Analysis of Interaction in the Case of No Replication in a Two-Factor Experiment

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Abstract :

This research was concerned with the problem of interaction between effects in the data that can be arranged in two directions, which are called two-way classification models , in the absence of redundancy in the observations . The research was applied to a set of two -way classification data and used two methods to test the presence of interaction in the model , namely the Tukey test, which depends on one degree of freedom, and the Mandel test (a stack of lines) . The data included five levels of phosphorus and three levels of nitrogen in the wheat crop ; it can be seen in appendix A .

As a result , the Tukey and Mandel tests have a significant effect on nesting. The interaction effect had a clear and significant effect on the results of the analysis and the main effects tests .

Keywords : Parameter Estimates , Randomized Complete Block Design , Tukey's test , Mandel test , Classification , ANOVA table .

مجلة الادارة والاقتصاد
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1. Introduction:

Statistics is a science that is presently used to analyze and draw conclusions for numerous research and studies in various fields. It is a set of logically based procedures, tools, rules, and laws that are utilized as the best way to measure and analyze occurrences and facts, conclude, and interpret them to elucidate the existing link between them. When performing statistical tests on proposed hypotheses or setting confidence limits on estimates, we use different distributions of samples that are determined purely mathematically by developing a model, assuming certain conditions about the model, and then follow the steps for the distribution of samples that fit the subject mathematical model. In the two- way classification model generated by having one observation per cell, the null hypothesis of main effects is tested using the M.S ratio. For the main effects to have the mean square error (MSE), but if there is an overlap between the main effects, this test is not valid, so the research focused on studying the composition of the overlap and the effect of the overlap in the usual test of the main effects. Fisher has the first credit for the development of factorial experiments and their analysis, Yates is a great credit for promoting this development and analysis of factorial experiments, as the work in factorial designs was developed by Yates in a pamphlet published in 1937; and in 1955, Tukey showed that his test could be extended to test the interference of the Latin square. There are many modifications suggested for Tukey's test, such as (1962) Tukey Mandel (1969) (1961) (1959), Harter (1962), and Millikan (1970), and all of these tests showed good power when the interference is a function of effects. In 1969, Mandel defined several models as special cases of the factorial analysis of variance model, and these cases were obtained by imposing a specific structure of interference η_{ij} in the general model. The applied studies on classification models of the two-way continued with the presence of interference in the case of recording one observation per cell to this day.

2. Objective

The research objective is to investigate the influence of interference on standard tests for its main effects and to identify one or more interfering factors.

3. Theoretical Aspect

3.1 Experiment Unit

The experimental unit is defined as the smallest part or section of the experimental materials to which the treatment is distributed in the experiment . It is used to record observations and measure the effect of coefficients on the variable under study . The experimental unit may be a human being , a plant , an animal , or a plot of land . (Aylin. 2006).

3.2 Treatments Unit

Treatment is defined as a set of experimental conditions that are placed under the control of the researcher, and the experimental units are distributed according to the chosen experimental design. The transactions under study may represent quantitative or descriptive transactions.

3.3 Experimental Error

It is a measure of the natural difference between the experimental units that were treated with the same treatment.

3.4 Analysis of Variance

It measures the natural difference between experimental units that experienced the same treatment . This error arises by subjective discrepancies between heterogeneous experimental units or by differences in the incorrect implementation of recurring transactions on the experimental units. Technical problems in recording or measuring observations might cause experimental errors.

3.5 Randomized Complete Block Design (RCBD)

It is that design in which the experimental units are divided into groups, each of which includes homogeneous experimental units within each sector, and the transactions are distributed randomly and independently within each sector, and each sector must contain all the transactions. The mathematical model for this design is:(anderson, 1974: morten, 2011).

$$y_{ij} = \mu + T_i + B_j + \varepsilon_{ij} \qquad (1)$$

Where:

μ : is the overall (grand) mean, T_i is the effect due to the i th treatment, βj is the effect due to the j th block, and ε_{ij} is the error term where the error terms, are independent observations from an approximately Normal distribution with mean = 0 and constant variance σ^2. Total variability of all of the Y_{ij}, . Which can be broken up into three parts: SST = SSTr + SSB + SSE

Table (1) ANOVA Table of Randomized Complete Block Design

S.O.V.	d.f.	S.S.	M.S.	F-stat
Blocks	r-1	$\frac{\sum Y.j^2}{t} - \frac{Y..^2}{rt}$	SSR/r-1	F=MSTr/MSE
Treatments	t-1	$\frac{\sum_i Y_i.^2}{r} - \frac{Y..^2}{rt}$	SSTr/t-1	
Error	(r-1)(t-1)	SSE=SST-SSR-SSTr	SSE/(t-1)(r-1)	
Total	rt-1	$\sum\sum y_{ij}^2 - \frac{Y..^2}{rt}$		

Again, the test of a treatment effect H0: μ1 = μ2 =…= μt HA: at least one mean differs, using the statistic F*=MST/MSE. If the null hypothesis is true then F* has an F-Distribution on numerator degrees of freedom t – 1 and denominator degrees of freedom (t – 1) (b – 1). In addition to the similarity of the F-test of equality of treatment means, the tests and comparisons of treatment means are done the same as before as well.

3.6 Interaction of Two-Way Classification

The inability of the levels one of the variables to retain the same degree and amount of effectiveness or efficiency at each level of the second component is referred to as factor nesting. The function of the two variables f (x, z) was used to define the concept of interaction. The model of two-way classification without nesting can be written in equation 2. (graybill, 1976: John ,2023).

$$y_{ij} = \mu_{ij} + e_{ij} \qquad (2)$$

i=1,2,..,t and j=1,2,..,b, where μij is the total effect of combinations of level i of factor A and level j of factor B.

So the total effect is just the sum of the effects i of A which is Ti the effect j

$$\mu_{ij} = \mu + T_i + B_j$$

$$\mu_{1j}- \mu_{2j} = T_1-T_2 \ , \ \mu_{1j'}- \mu_{2j'} = T_1-T_2$$

That leads to that

$$(\mu_{1j}- \mu_{2j}) - (\mu_{1j'}- \mu_{2j'}) = 0$$

in general

$$(\mu_{ij}- \mu_{i'j}) - (\mu_{ij'}- \mu_{i'j'}) = 0$$

for every j', j , i', i

3.7 Nesting Test

Many researchers have studied the nesting problem in the two-way classification model, with one view of each cell, and presented many methods of this to test including:

3.7.1 Tukey Test

The following method is to test for nesting in the two-way classification to any hypothesis.The following method is to test for nesting in the two-way classification of any hypothesis Tukey.(Tukey,2002).

$$H_0 : \eta=0 \ \text{vs.} \ H_A : \eta \neq 0$$

a. Model Tukey Test depends on assuming that. (Tukey, 2002).

$$\eta= \eta_{ij}=G\, \alpha_i\, \beta_j \qquad (3)$$

Where G is a constant, i.e. the nesting ηij is a function of the main effects αi and βj of the cell. This function is assumed to be a square polynomial. (Tukey, 2002).

$$\eta_{ij}=A+B\alpha_i+C\beta_j+D\, \alpha_i^{\,2}+G\alpha_i\, \beta_j +H\beta_j^{\,2} \qquad (4)$$

The $\alpha_. = \beta_. = \eta_{i.} = \eta_{.j} = 0$ in equation 4 can calculate $\eta_{i.} = A+ B\alpha_i +D\alpha_i^{\,2}+H\theta=0$, when $\theta=\Sigma \beta j2/J$

and $\eta_{.j} = A+ C\beta_j+D\Phi+H\beta_j^2=0$ while $\Phi= \Sigma\, \alpha_i^{\,2} /I$ then $B\alpha_i +D\alpha_i^{\,2}=-A- H\theta$, $C\beta_j+ H\beta_j^2=-A-D\Phi$ putting in equation 4 we can getting to

$$\eta_{ij}=-A- H\theta- D\Phi+ G\alpha_i\, \beta_j \qquad (5)$$

But $\eta_{i.} =-A- H\theta- D\Phi=0$ then the model is equal to equation 6, where $\varepsilon_{ij} \sim N(0, \sigma^2\varepsilon)$ and

$$y_{ij} = \mu + \alpha_i + \beta_j +G\, \alpha_i\, \beta_j + \varepsilon_{ij} \qquad (6)$$

b. ANOVA Tabel for Nesting Test

In testing for Nesting in a two factor design with one observation per cell

Table (2) ANOVA Table of two factor Design with one Observation per cell

S.O.V.	d.f.	S.S.	M.S	F-stat
Factor A	a-1	SS_A		
Factor B	t-1	SS_T		
No additivity	1	SSG	MSG	FG
Residual	(a-1)(t-1)-1	SSR	MS_R	
Total	ta-1	$\sum Y_{ij}^2- Y..^2/ta$		

$$SS_G=[\Sigma\Sigma y_{ij}(y_{i.}-y_{..})(y_{.j}-y_{..})]^2 / \Sigma(y_{i.}-y_{..})^2\Sigma(y_{.j}-y_{..})^2 \qquad (7)$$

While $[\Sigma\Sigma y_{ij}(y_{i.}-y_{..})(y_{.j}-y_{..})]^2= [\Sigma\Sigma y_{ij}y_{i.}y_{.j}/at -(y_{..}/at)((\Sigma y_{i.}^2/t - y_{..}^2/at) + (\Sigma y_{.j}^2/a - y_{..}^2/at) +y_{..}^2/at)]^2 = 1/a^2t^2\, [\Sigma\Sigma y_{ij}y_{i.}y_{.j} - y_{..}(Ass+Tss+y_{..}^2/at)]^2$

3.7.2 Mandel Test

It is using ANOVA table in the two-way classification model of with one observation per cell.

a. Mandel Model Test in (1961) Mandel proposed a Stack model of straight lines and used the ANOVA table and type nesting test is **$\eta_{ij} = Q_i\gamma_j$.** Need to be associated with the main influences. Let y_{ij} be observations classified according to two criteria: **A_i and B_j, where (j = 1, 2,..., n, i = 1, 2,...,m.** We will assume that the errors of y_{ij} observations are a sample from a natural population with mean of zero and a variance of σ^2, and a the model is :(Mandel.1969).

$$y_{ij}=\mu+\rho_i+\gamma_j+T_{ij}+\varepsilon_{ij} \quad (8)$$

$$\Sigma\rho i=\Sigma\gamma j=0 \quad (9)$$

(10)Where: $\bar{y}_{.j}=\sum_i y_{ij}/m$ **, $E(y_{ij})=\mu+\rho_i+\gamma_j+T_{ij}$ and $E(\bar{y}_{.j})=\mu+\gamma_j$.**

This is give as the good properties of nest it can be seen in equation 11. (Mandel,1969).

$$E(y_{ij}-\bar{y}_{.j})=\rho_i+T_{ij} \quad (11)$$

When $\Sigma\rho_i=\Sigma\gamma_j=\Sigma Q_i=0$, the model can be written in equation 12. (Mandel,1969).

$$y_{ij}=\mu+\rho_i+\gamma_j+Q_i\gamma_j+\varepsilon_{ij} \quad (12)$$

b. ANOVA Tabel for Nesting Test

When using ANOA test depends Mandel model in two way classification model (Snee, 1982).

Let $\bar{y}=\bar{y}_{..}$ **,** $R_i=\bar{y}_{i.}-\bar{y}_{..}$ **and** $C_j=\bar{y}_{.j}-\bar{y}_{.}$ **We get**

$$y_{ij}=\bar{y}+R_i+C_j+(b_i-1)C_j+\Delta_{ij} \quad (13)$$

Where $\Delta_{ij}=(y_{ij}-y_{i.})-b_iC_j$

$$\Sigma\Sigma y_{ij}^2=mn\,\bar{y}^2+n\Sigma R_i^2+m\Sigma C_j^2+\Sigma(b_i-1)^2\Sigma C_j^2+\Sigma\Sigma\,\Delta_{ij}^{\ 2} \quad (14)$$

ANOVA table can be shown e in Table (3).

Table(3) ANOVA Table for Identical Data

S.O.V	d.f.	S.S
Rows	m-1	$n\sum R_i^2$
Columns	n-1	$m\sum C_j^2$
Slopes	m-1	$\sum$ (b$_i$-1)2 $\sum$ Cj2
Residual	(m-1)(n-2)	$\sum\sum\Delta_{ij}^{\ 2}$
Total	mn-1	$\sum\sum y_{ij}^2-mn\,\bar{y}^2$

When comparing equations 13 and 14 it can see that Qiγj and εij are the same with (bi-1)Cj and Δij.

Let Qi+1=Bi,it means that equation13 can be written as: (Snee, 1982).

$$y_{ij}=\mu+\rho_i+\gamma_j+(B_i-1)\gamma_j+\varepsilon_{ij} \quad (15)$$

If $\mu_i=\mu+\rho_i$ we get to this equation

$$y_{ij}=\mu_i+B_i\gamma_j+\varepsilon_{ij} \quad (16)$$

4. Practical Aspect

The data included five levels of phosphorus and three levels of nitrogen in the wheat crop.

a. Two-Way Classification Model

Description of the data collection methods depends on equation 1 to analyze this data where **i=1,2,3 and j=1,2,3,4,5.**

Where: yij: the observation value of level i of nitrogen and level j of phosphorus.

μ: the general mean value.

Ti: effect of the i level of nitrogen.
Bj: the effect of the j level of phosphorus.
ij: the effect of the experimental error of the observation of level i of nitrogen and level j of phosphorus.

b. ANOVA Table of Randomized Complete Block Design

After analyze the data, we get this result it can be shown in Table (4).

Table (4) ANOVA Table of Randomized Complete Block Design

Source	df	SS	MS	F	Fα(0.05)
Nitrogen	2	328075.6	164037.8	50.731	4.46
Phosphorus	4	3748569.07	937142.26	28.985	3.84
Residual	8	258651.73	32331.466		
Total	14	4335296.40			

From the results of the ANOVA table, it was found that there were significant effects of phosphorous and nitrogen on the amount of wheat production.

c. Tukey Model Test

To find out the presence of interference between the treatments (nitrogen and phosphorus), we applied the Tukey test, which depends on one degree of freedom. The results are shown in Table (5)

Table (5) Shows the Result of Tukey Test

Source	Df	SS	MS	F	Fα(0.05)
Nitrogen	2	328075.60			
Phosphorus	4	3748569.07			
Non-additivity	1	235766.56	235766.56	72.11	5.59
Residual	7	22885.17	3269.31		
Total	14	4335296.30			

The calculated F value for the nesting shown in Table 5 is 72.11, which is greater than the table value 5.59 at a significant level of 0.05; we conclude that there is a significant effect of the nesting.

d. Mandel Model Test

By depending on equation 6 for using Mandel model test to find this value

b₁= $\frac{\sum y_{ij}C_j}{\sum C_j^{\,2}} = \frac{808349.036}{1249521.965} = 0.6469$

b₂= $\frac{150.9660.819}{1249521.965} = 1.2081907$

b₃= $\frac{1430566.007}{1249521.965} = 1.14489$

Slope SS= $\sum (b_i - 1)^2 \sum C_j^{\,2} = 236156\,.2571$

Table (6) Shows the Result of Mandel Test

Source	D.F.	S.S.	M.S.	F.	Fα(0.05)
Nitrogen	2	328075.6			
Phosphorus	4	3748565.894			
Slopes	2	236156.251	118078.128	31.498	5.14
Residual	6	22495.4661	3749.244		
Total	14	4335296.30			

By calculating the value of F for slope and the table value of F with degrees of freedom 2 and 6, it is clear that there is a significant effect.

Table (7) Shows the Sum Square of the Slope

Source	D.F	S.S	M.S.	F.	Fα(0.05)
Slopes	2	236156.25			
Concurrence	1	235772.12	235772.1	613.7	161.4
Non-concurrence	1	384.131	384.131	0.102	5.99

The result shown in Table (7) shows the calculated values of F for the intersection, which represents the nesting. It turns out that it has a significant effect, while it was found that there is no effect of the non-intersection.

5- Conclusions & Recommendations

5.1 Conclusions

After applying the Tukey and Mandel test to interaction in the case of no replication of the data to using five levels of phosphorus at three levels of nitrogen in the wheat crop in southern Kurdistan, the conclusions reached are found as:

1. It can be shown in the application part that it was concluded that the Tukey and Mandel tests have a significant effect on nesting.
2. The data show that the interaction replication in a two-factor experiment using a randomized complete block design has a significant effect.
3. By using the sum square of the slope calculated F for the intersection, there is a significant effect and no effect for the non-intersection.
4. The interaction effect had a clear and significant effect on the results of the analysis and the main effects tests.

5.2 Recommendations

After analysis and conclusions, the researcher recommends the following:

1. It is necessary to detect the presence of interference in the two-way classification model by using the Tukey and Mandel test.
2. Depending on this test to find an appropriate estimate of the error variance
3. Try to find an appropriate estimate of the error variance.
B. Define the interaction shape to see if one or more parameters or one or more views are causing the interaction.

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تحليل التفاعل في حالة عدم التكرار في تجربة ذات عاملين

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المستخلص :

اهتم هذا البحث بمشكلة التداخل بين التأثيرات في البيانات التي يمكن ترتيبها باتجاهين، والتي تسمى بنماذج التصنيف باتجاهين، في حالة عدم التكرار في المشاهدات. تم تطبيق على مجموعة من بيانات التصنيف ذات التصنيف باتجاهين واستخدم طريقتين لاختباروجود التداخل في النموذج وهما اختبار توكي الذي يعتمد على درجة الحرية واحدة واختبار ماندل (رزمة من الخطوط). تضمنت البيانات خمسة مستويات من الفوسفور وثلاثة مستويات من النيتروجين في محصول القمح، ويمكن رؤيتها في الملحق أ.

ونتيجة لذلك، كان لاختبارات توكي وماندل تأثير كبير على التداخل وكان تأثيرا واضح ومعنوي على نتائج التحليل واختبارات التأثيرات الرئيسية.

الكلمات المفتاحية : تقدير المعلمات ، تصميم القطاعات العشوائية الكاملة ، اختبار توكي ، اختبار ماندل ، التصنيف ، جدول تحليل التباين .

Appendix (A) Wheat Crop Data

Nitrogen in kg/ha	0	22	Phosphorus in kg/ha 45 90		180	Total
0	1984	2550	2706	2740	2954	12934
45	1776	2843	3306	3305	3386	14616
90	1797	2761	3240	3227	3332	14357
Total	5557	8154	9252	9272	9672	41907

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