A Study and Estimation OF Reliability Function of Kumaraswamy Perks Distribution

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Abstract

In this paper some of Kumaraswamy Perks distribution properties probability density function, Reliability function, hazard function and others were discussed. The maximum likelihood, shrinkage and LSEM method are used for estimating the reliability function based on complete sample which was obtained from the simulation using matlab programs. The results of simulation showed convergence of reliability values .

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1-Introduction

Classical statistical distributions have been used to describe random behavior of any phenomenon in different fields such as engineering, medicine, agriculture, science, demography and others. But in many situations these distributions are not suitable to describe the random behavior of the phenomenon under study. This prompted[1] many researchers to pay more attention to find the probability distribution which is flexible and more accurate in describing random behavior of such phenomena , by adding new parameters to the classical probability distributions that makes them more flexible to describe random behavior of a phenomenon. Many [(8)] methods were suggested to find new class of probability distributions. Kumaraswamy [(6)] suggested a probability distribution known as Kumaraswamy distribution, for which the probability cumulative distribution function is [(7)]:

$$F(x) = 1-\left(1-x^{\alpha}\right)^{\beta} \quad 0 < x < 1; \alpha > 0; \beta > 0 \qquad (1)$$

Where α and β are $shape$ parameters.

Eugene [(5;6;8)] et al. has proposed a method for obtaining a new class of distributions based on the beta distribution and has been named the beta-generator family. Cordeiro and Castro [(4)] also suggested another class of distributions called Kumaraswamy generalized distribution by merging the methods of Eugene and Kumaraswamy to obtain a new class of distribution according to the following formula [(10)]

$$F(x) = 1-\left(1-G(x)^{a}\right)^{\beta} \qquad (2)$$

Where $G(x)$ is the cumulative distribution function and $(\alpha;\beta)$ are additional parameters. The density distribution function for Cordeiro and Castro can be obtained by driving equation (2) as follows:

$$f(x) = \alpha\beta g(x) G(x)^{\alpha-1}\left(1-G(x)\right)^{\beta-1} \qquad (3)$$

2-Perks distribution

Perks [(4)] distribution is extension to Gompertz–Makeham distribution and its name came from the researcher's name who created it. Perks distribution is one of the important distributions which is applied to actuarial science that includes models for pensioner mortality data. The probability cumulative distribution function of parks distribution has two parameters that can be written as [(10)]

$$F(x,\gamma,\theta) = 1-((1+\theta)/(1+\theta e\hat{\ }\gamma x\ )) \quad x > 0; \gamma; \theta \geq 0 \qquad (4)$$

Where γ is shape parameter,θ is scale parameter. The probability density function of perks distribution is

$$f(x;\gamma;\theta) = \gamma\theta e^{\gamma x}\,\frac{1+\theta}{(1+\theta e^{\gamma x})^2} \qquad (5)$$

And the reliability function is

$$R(t,\gamma,\theta) = \frac{1+\theta}{(1+\theta e^{\gamma t}\ )} \qquad (6)$$

The hazard function is

$$h(x) = \frac{\theta\gamma e^{\gamma x}}{(1+\theta e^{\gamma t})} \qquad (7)$$

This distribution doesn't have moments function in a closed form [(3)]

(1) View (7)

2-Kumaraswamy Perks Distribution (KW-P)

If the probability cumulative distribution function. Eq (4) of Perks distribution substituted in equation (2), kumaraswamy perks distribution (KW-P) will result. The cdf of the KW-P is

$$F(x;\alpha;\beta;\theta;\gamma) = 1-\left(1-\left(1-\frac{1+\theta}{(1+\theta e^{\gamma x})}\right)^{\alpha}\right)^{\beta} \qquad (8)$$

Where $(\alpha;\gamma)$ are shape parameters $(\beta;\theta)$ are scale parameters. The pdf of this distribution will be as

$$f\ (x;\alpha;\beta;\theta;\gamma)=\alpha\beta\theta\gamma e^{\gamma x}\ \frac{1+\theta}{\left(1+\theta e^{\gamma x}\right)^{2}}\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha-1}\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta-1} \qquad (9)$$

Plots of the probability density function for selected parameters values are given in figure (1). The reliability function of kw-p is

$$R\ (t;\alpha;\beta;\theta;\gamma)=\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta} \qquad (10)$$

The hazard function means that the probability of in item survives until time (t) and falls after (Δt) so

$$h(t)=\frac{f(t)}{(1-F(t))}=\frac{f(t)}{R(t)} \qquad (11)$$

For kw-p distribution the hazard function will be

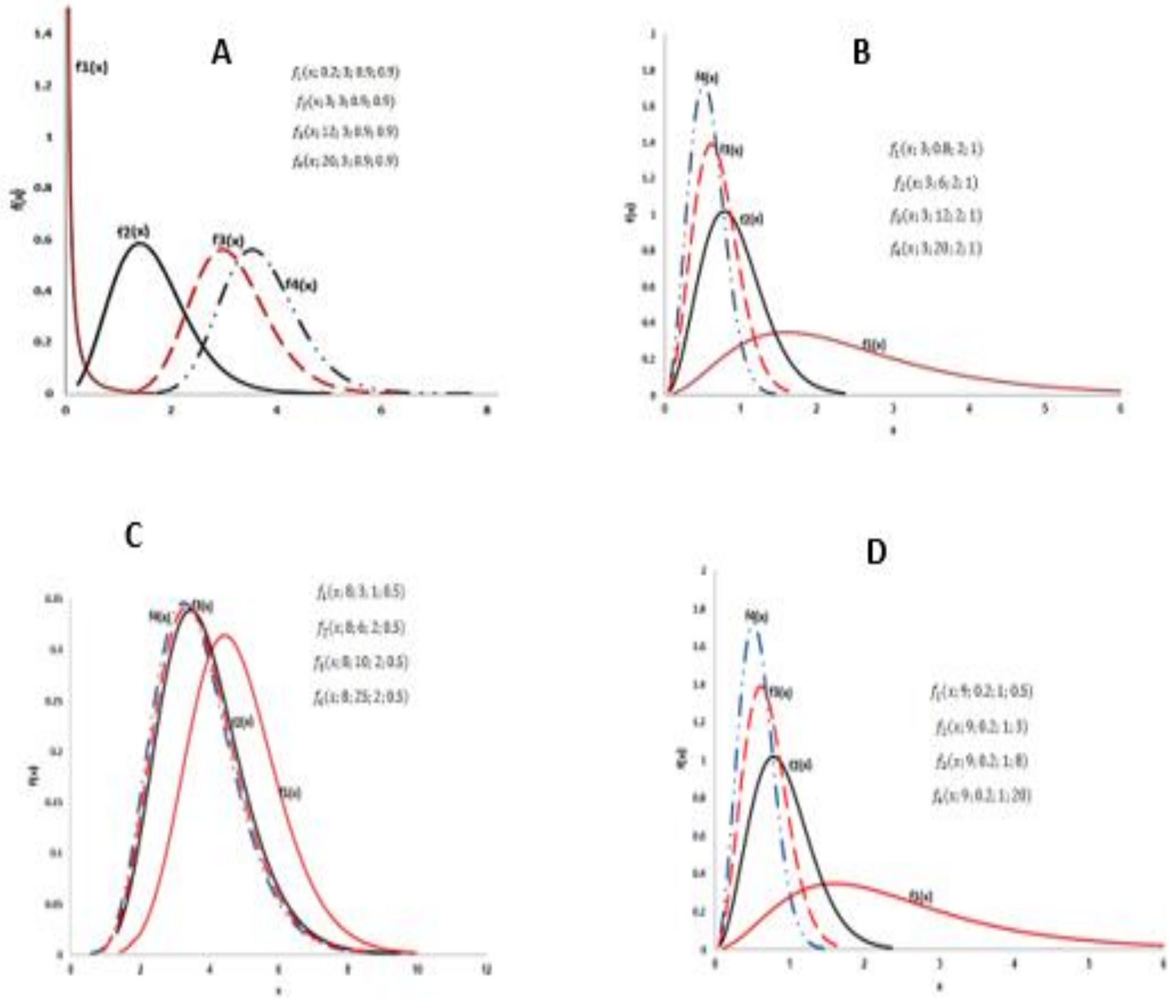

Fig (1): probability density function of KWP distribution: (A) different value of (α)

(B): different value of (β) : (C): different value of (θ) (D): different value of (D)

$$h(t)=\alpha\beta\theta\gamma e^{\gamma x}\,\frac{1+\theta}{\left(1+\theta e^{\gamma x}\right)^{2}}\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha-1}\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{2\beta-1} \qquad (12)$$

The cumulative hazard function of KW-P can be written as

$$H(t)=-\log(1-F(t))$$

$$H(t)=-\log\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta} \qquad (13)$$

The failure rate average is

$$FAT=\frac{H(t)}{t}$$

And for kw-p is

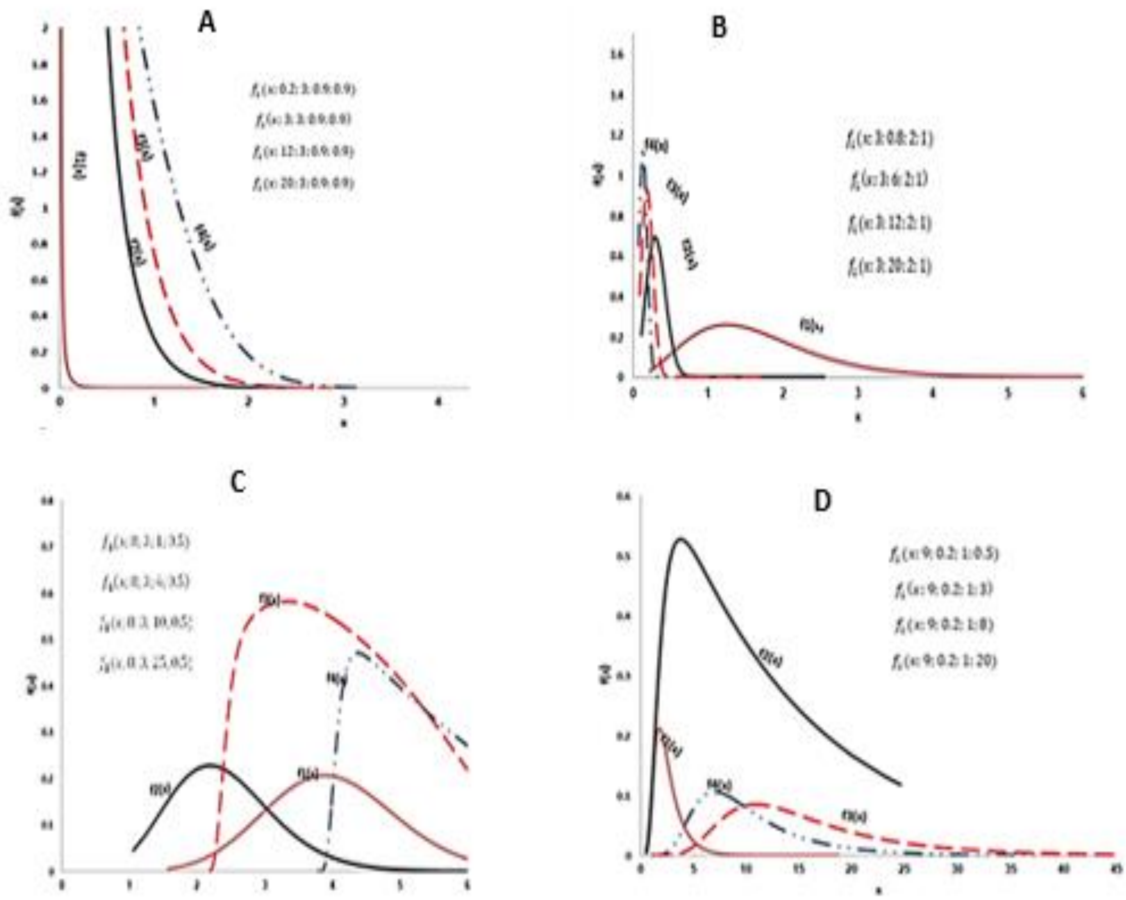

Fig2: Hazard function of KWP distribution for: A. different value of (α)

B) different value of (β) C) different value of (θ) D) different value of (γ)

$$FAT = -\frac{\log\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta}}{t} \qquad (14)$$

Plots of hazard function of KW-P distribution for selected parameters is shown in figure (2)

4-The Properties of KW-P Distribution

In this section some of important properties of KW-P distribution will be mentioned.

We can't give a close form for the moments of KW-P distribution since the integration in equation (4-1) doesn't have a closed form

$$E(x^2) = \alpha\beta\theta\gamma \int_{x=0}^{\infty} x^k e^{\gamma x} \frac{1+\theta}{(1+\theta e^{\gamma x})^2} (1-\frac{1+\theta}{1+\theta e^{\gamma x}})^{\alpha-1} (1-(1-\frac{1+\theta}{1+\theta e^{\gamma x}})^{\alpha})^{\beta-1} \qquad (15)$$

The mode is a value of (x) which maximize the following equation

$$f^{'}(x)=\left[\left(\alpha\beta(1+\theta)e^{\gamma x}-1-\theta\left(e^{\gamma x}\right)^{2}\right)\left(\frac{\theta\left(e^{\gamma x}-1\right)}{1+\theta e^{\gamma x}}\right)^{\alpha}+1+\theta e^{2\gamma x}-\alpha(1+\theta)e^{\gamma x}\right]\alpha\beta\theta\gamma^{2}e^{\gamma x}(1+\theta)$$

$$-\left(1-\frac{\theta\left(e^{\gamma x}-1\right)}{1+\theta e^{\gamma x}}\right)^{\beta-1}\left(\frac{\theta\left(e^{\gamma x}-1\right)}{1+\theta e^{\gamma x}}\right)^{\alpha-1}$$

The median of KW-P distributions is given by (16)

$$x_{0.5}=-\frac{1}{\gamma}\left(ln(\theta)-ln\left(1-\left(1-2^{-\frac{1}{\beta}}\right)^{\frac{1}{\alpha}}\right)-ln\left(\theta+\left(1-2^{-\frac{1}{\beta}}\right)^{\frac{1}{\alpha}}\right)\right) \quad (17)$$

The quintile function of KW-P distribution is:

$$x_{p}=1-\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta} \quad (18)$$

The KW-P distribution's order statistics is as follows

$$g_{k}(y_{k})=\frac{n!}{(k-1)!(n-k)!}\alpha\beta\theta\gamma\frac{(1+\theta)^{2}}{\left(1+\theta e^{\gamma x}\right)^{2}}\left(1-\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta}\right) \quad (19)$$

$$\left(\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta}\right)^{n-k}\left(1-\left(\frac{1+\theta}{1+\theta e^{\gamma x}}\right)\right)^{\alpha-1}\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)^{\beta-1}$$

5-Estimation Methods

The methods of Maximum likelihood estimator and Shrinkage estimator were used to estimate parameters of KW-P distribution

5-1 Maximum Likelihood Method

It's one of the most important estimation methods. An estimate can be found for the reliability function of KW-P distribution by finding the parameters of probability distribution function by using invariant properties. The calculations of maximum likelihood estimator method were based on complete sample ($x_1; x_2 \ldots; x_n$) with KW-P distribution. The log-likelihood functions are

$$\mathrm{L}=nln(\alpha)+nln(\beta)+nln(\theta)+nln(\gamma)+\varphi(x,\alpha,\beta,\theta,\gamma) \quad (20)$$

Where

$$\varphi(x,\alpha,\beta,\theta,\gamma)=\sum_{i}^{n}\ln\left(e^{\gamma x_i}\frac{(1+\theta)}{\left(1+\theta e^{\gamma x_i}\right)^{2}}\right)+(\alpha-1)\sum_{i}^{n}ln\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)$$

$$-(\beta-1)\sum_{i}^{n}ln\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x}}\right)^{\alpha}\right)$$

To obtain an estimation for the parameters ($\alpha;\beta;\theta;\gamma$), equation (21) must be maximized with respect to ($\alpha;\beta;\theta;\gamma$). So,

$$\frac{\partial \mathrm{L}}{\partial \alpha}=\frac{n}{\alpha}+\sum_{i}^{n}\ln\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)+(\beta)\sum_{i}^{n}\frac{\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)\ln\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)}{1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)} \quad (22)$$

$$\frac{\partial \mathrm{L}}{\partial \beta}=\frac{n}{\beta}-\sum_{i}^{n}\ln\left(1-\left(1-\frac{1+\theta}{1+\theta e^{\gamma x_i}}\right)^{\alpha}\right) \quad (23)$$

$$\frac{\partial \mathrm{L}}{\partial \theta}=\frac{n}{\theta}\sum_{i}^{n}\frac{1-(\theta+2)e^{\gamma x_i}}{(1+\theta)\left(1+\theta e^{\gamma x_i}\right)}+(\theta+1)\sum_{i}^{n}\frac{1}{\theta\left(1+\theta e^{\gamma x_i}\right)}$$

$$-(\beta-1)\sum_{i}^{n}\frac{\alpha\theta^{\alpha-1}\left(e^{\gamma x_i}-1\right)\left(1+\theta e^{\gamma x_i}\right)^{-(\alpha+1)}}{\theta^{\alpha}\left(e^{\gamma x_i}-1\right)^{\alpha}\left(\theta e^{\gamma x_i}+1\right)-1} \quad (24)$$

$$\frac{\partial \mathrm{L}}{\partial \gamma}=\frac{n}{\theta taal\,\gamma}\sum_{i}^{n}\frac{\left(\theta e^{\gamma x_i}-1\right)}{\left(1+\theta e^{\gamma x_i}\right)}+(\theta+1)\sum_{i}^{n}\frac{(1+\theta)x_i e^{\gamma x_i}}{\left(1+\theta e^{\gamma x_i}\right)\left(e^{\gamma x_i}-1\right)} \quad (25)$$

$$-(\beta-1)\sum_{i}^{n}\frac{\alpha\theta^{\alpha}\left(e^{\gamma x_i}-1\right)^{\alpha-1}\left(1+\theta e^{\gamma x_i}\right)^{-(\alpha+1)}(1+\theta)x_i e_i^{\gamma x}}{\theta^{\alpha}\left(e^{\gamma x_i}-1\right)^{\alpha}\left(\theta e^{\gamma x_i}+1\right)-1}$$

Equalization of equations (18 to 21) to zero will lead to a nonlinear system of equations, which are solved by using numerical methods.

5-2 Shrinkage Methods:

The shrinkage methods for KW-P distribution is represented by(12;13;14)

$$\mathrm{R_{sh}} = w\widehat{R} + (1-\mathrm{w})\mathrm{R_0} \quad (26)$$

Where

R_0: represents initial value for reliability of KW-P distribution calculated using defaulted parameters $(\alpha;\beta;\theta;\gamma)$.

$\hat{R}$: estimates value for reliability of KW-P distribution.

w: shrinkage factor, which is a constant value specified according to (R_0).

In general, the value of (w) falls in the period $(0 < w < 1)$ and chosen to minimize mean square error.

The value of (w) which minimizes mean square error is

$$w = \frac{(R_0 - R)^2}{(mse(\hat{R}) + (R_0 - R)^2)} \quad (27)$$

$MSE(\hat{R})$ represent the values of mean square error of estimated reliability

6-Least Square Error Methods

This method is based on minimizing sum of square errors resulting from the difference between observations of samples and expected values according to linear model bellow.

$$u_i = y_i - \beta_0 - \beta_1 x \quad (28)$$

The reliability function of KW-P distribution is given by

$$R = \left(1 - \left(1 - \frac{1+\theta}{1+\theta e^{\gamma t_i}}\right)^{\alpha}\right)^{\beta} \quad (29)$$

By taking the logarithm for equation (29) the following results is obtained.

$$\ln\left(1 - R^{\frac{1}{\beta}}\right) = \alpha\ln(\theta) + \alpha\ln\left(\frac{e^{\gamma t_i}-1}{1+\theta e^{\gamma t_i}}\right) \quad (30)$$

Comparing equations (28) and (30)

$y_i = \ln\left(1 - R^{\frac{1}{\beta}}\right)$; $x_i = \ln\left(\frac{e^{\gamma t_i - 1}}{1+\theta e^{\gamma t_i}}\right)$; $\beta_0 = \alpha \ln(\theta)$; $\beta_1 = \alpha$

The estimation of reliability, according to Least square error methods, is[2] given by

$$\hat{\alpha} = \frac{n \sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right) \ln\left(1 - R^{\frac{1}{\beta}}\right) - \sum_i \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right) \sum_i \ln\left(1 - R^{\frac{1}{\beta}}\right)}{\left(n \sum_i \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right)\right)^2 - \left(\sum_i \ln\left(1 - R^{\frac{1}{\beta}}\right)\right)^2} \quad (31)$$

$$\theta = \exp\left(\frac{1}{\alpha}\left(\frac{\ln\left(1 - R^{\frac{1}{\beta}}\right)}{n} - \frac{\left(n\left(\sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right)\right) \ln\left(1 - R^{\frac{1}{\beta}}\right)\right) - \left(\sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right) n \ln\left(1 - R^{\frac{1}{\beta}}\right)\right)\left(\sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right)\right)}{n\left(\sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right)\right)^2 - \left(\sum_{i=1}^{n} \ln\left(\frac{e^{\gamma t_i} - 1}{1 + \theta e^{\gamma t_i}}\right)\right)^2}\right)\right) \quad (32)$$

$$\hat{\gamma} = \frac{\ln\left(-\frac{1 + e^t}{\theta e^{t_i} - 1}\right)}{t_i} \quad (33)$$

$$\hat{\beta} = \frac{\ln(R)}{\ln\left(1 - \left(\frac{\theta(e^{\gamma t_i} - 1)}{1 + \theta e^{\gamma t_i}}\right)\right)} \quad (34)$$

$R(t_i)$ is Empirical Distribution functions which is

$$R(t_i) = \frac{i - 0.5}{n} \quad (35)$$

7-Simulation:

Simulation has been used to analyze the behavior of KW-P distribution based on simulations of different samples with KW-P distribution. An inversion method has been used to obtain samples of data with KWP distribution, the algorithm for generating the samples is given by the following steps

1- Generating $u\_i \sim uniform(0,1) \quad i = 1,2, \dots n$

2- Set $u = F^{-1}(x)$

Where $F^{-1}(x)$ is invers function of cumulative function of KW-P distribution

The simulated samples were having the following sizes $(n = 15; 25; 35; 75; 100; 150)$ with kw-p distributions. The values of the chosen parameters are tabled as below:

Table (1): different parameters values selected for simulation

Model	α	β	θ	γ
Model1	0.2	3	0.9	0.9
Model 2	3	0.8	2	1
Model 3	8	3	10	0.5
Model 4	9	0.2	1	0.5

(2) View (2)

The results of simulation are compared with each other by using mean square error.

$$MSE = \frac{\sum_{i=1}^{k}\left(\hat{R}_i - R_i\right)}{k} \qquad (24)$$

$\hat{R}_i$ represents mean of estimated values of reliability and R_i represents mean of actual values of reliability.

7-Results

Monte Carlo simulation was used to obtain samples of different sizes with default parameters values for studying the random behavior of KW-P distribution as well as comparing methods of estimation which has been used in this study. Results of simulation were tabled in table (2) .The actual values and the estimated values by maximum likelihood, shrinkage and Least square methods of reliability were calculated for all default models and for all sizes of samples.

Table (2)

Average reliability estimator and MSE for three estimation methods

Model	n	R	Average reliability			Average Mean Square Error		
			MLM	SRHM	LSEM	MLM	SRHM	LSEM
Model1	15	0.31196	0.28446	0.39436	0.36643	0.16452	0.15950	0.12306
	25	0.58588	0.43311	0.61694	0.54095	0.14158	0.12126	0.01173
	35	0.42902	0.39934	0.45164	0.36407	0.13248	0.15083	0.04339
	75	0.44632	0.47176	0.49468	0.38485	0.01265	0.15234	0.02867
	100	0.63218	0.67176	0.49468	0.54825	0.01670	0.02199	0.01583
	150	0.50143	0.49322	0.48919	0.49615	0.00154	0.01673	0.00537
Model2	15	0.68819	0.50507	0.50740	0.62359	0.22633	0.10693	0.12659
	25	0.48079	0.62691	0.46036	0.47608	0.19660	0.15452	0.13509
	35	0.42902	0.39934	0.45164	0.36407	0.15936	0.11705	0.10556
	75	0.51076	0.50592	0.46112	0.47139	0.01554	0.13682	0.01762
	100	0.44090	0.44102	0.45933	0.51380	0.03336	0.13920	0.06766
	150	0.48573	0.48781	0.47741	0.39603	0.00485	0.01626	0.01430
Model 3	15	0.71895	0.57277	0.68966	0.71444	0.13922	0.16642	0.11165
	25	0.42335	0.44617	0.51119	0.42407	0.16578	0.12770	0.03379
	35	0.50232	0.56417	0.51221	0.50280	0.15156	0.14350	0.01616
	75	0.46793	0.46807	0.50345	0.49144	0.01063	0.19779	0.01796
	100	0.51677	0.51629	0.47303	0.54729	0.00115	0.01624	0.01213
	150	0.51063	0.51009	0.49522	0.49774	0.00311	0.01485	0.01913
Model 4	15	0.52157	0.50779	0.47076	0.52299	0.17917	0.22350	0.12787
	25	0.45784	0.58853	0.41820	0.45729	0.17811	0.17689	0.06278
	35	0.50192	0.53326	0.46879	0.50118	0.17403	0.10525	0.12944
	75	0.50565	0.50544	0.52628	0.45116	0.11942	0.12652	0.13058
	100	0.53565	0.52444	0.59205	0.45913	0.01535	0.02887	0.01746
	150	0.48827	0.48878	0.50557	0.53492	0.00140	0.01438	0.00339

It’s clear from table (2) that the results of simulation show convergence in MSE for all three estimation methods and for all default samples sizes and models. For small sizes of samples, method of LSE is better than MLE and SHRM because the average of estimated reliability is closer to the default reliability values. On the other hand, SHRM is better than MLE.

For big sizes of samples, the average of estimated reliability in method of MLE is closer to the default reliability values in compare with LSE and SHE methods. MSE Criteria showed the same results obtained above.
Fig-3(a-b-c-d-e-f) represented the curve of reliability of KW-P distribution

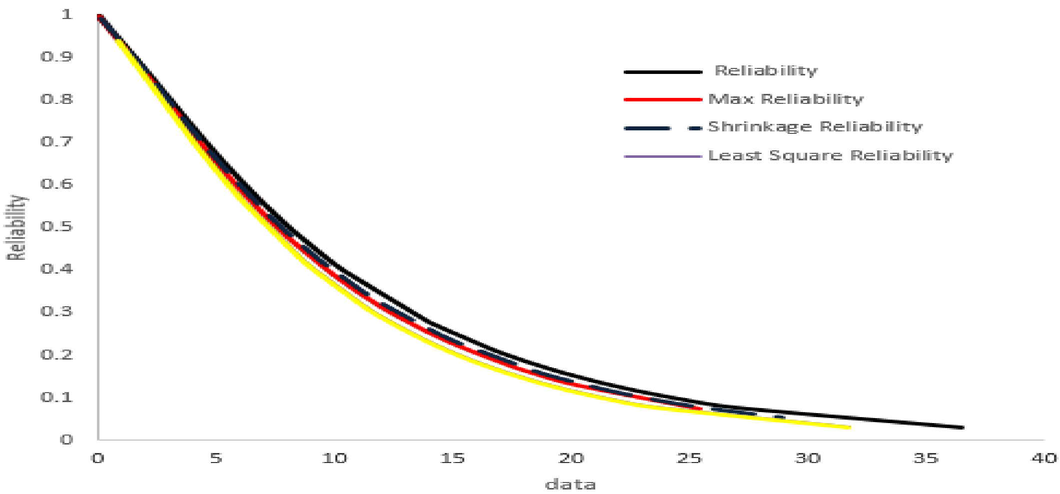

Fig.3-a- .Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 150)$

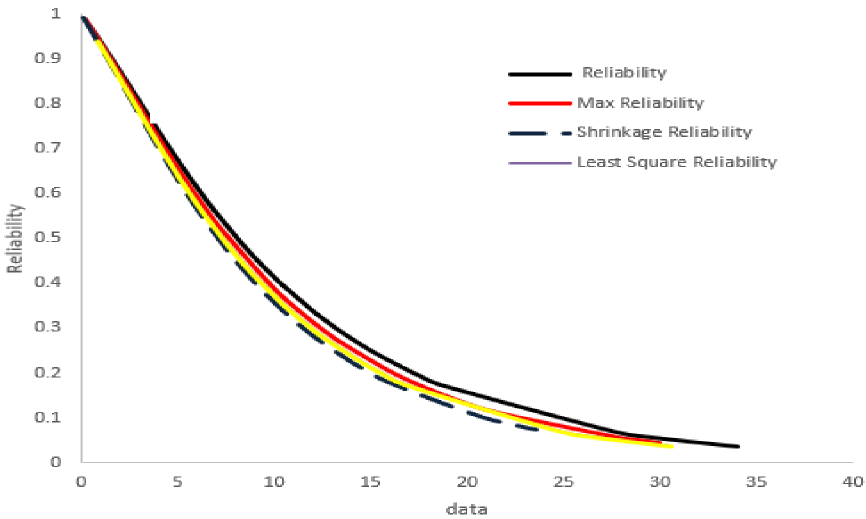

Fig.3-b- .Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 100)$

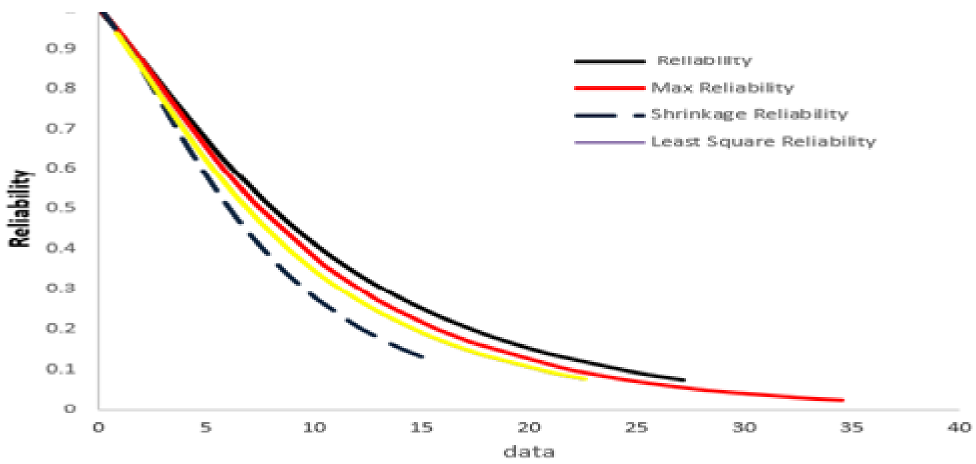

Fig.3-c-.Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 75)$

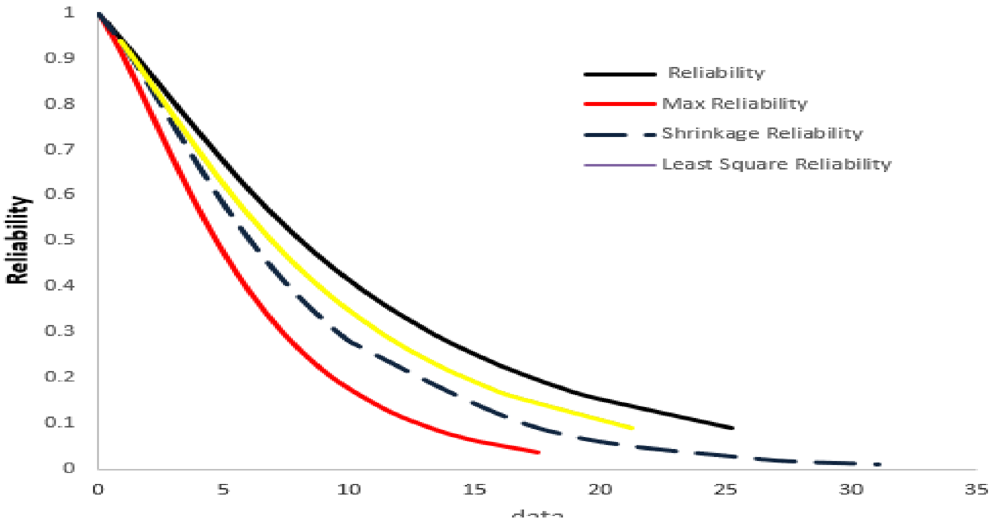

Fig.3-d- .Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 75)$

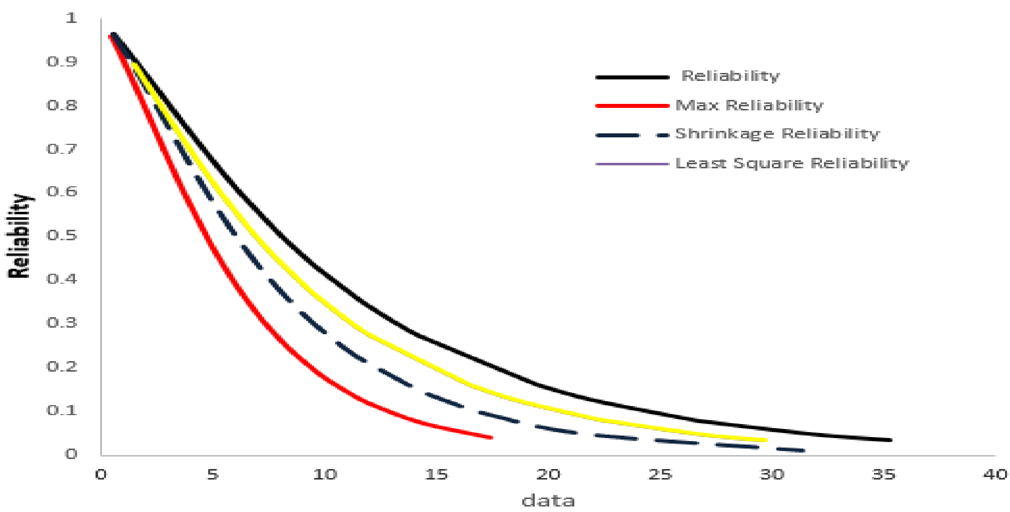

Fig.3-e- .Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 35)$

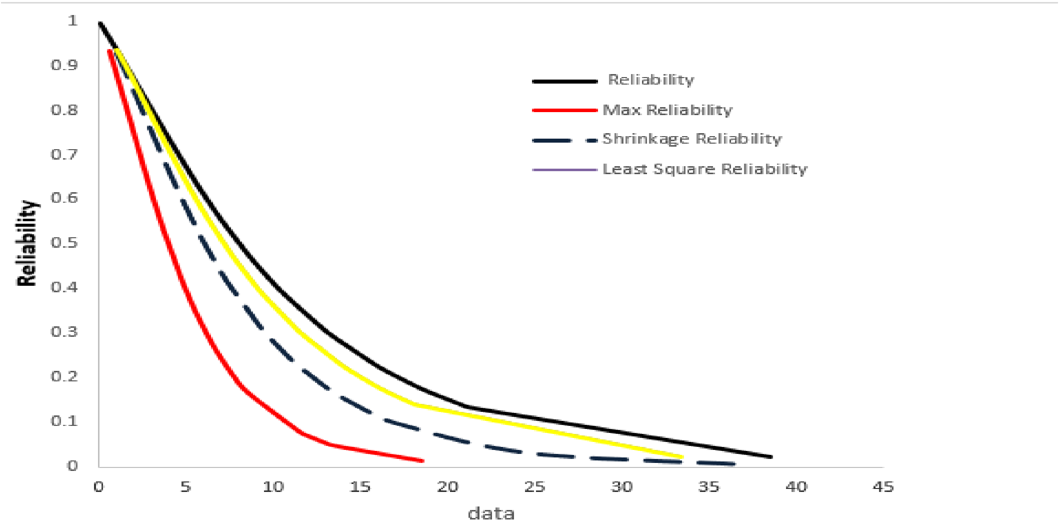

Fig.3-f- .Curve of reliability function of KWP distribution with parameters $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ and $(n = 25)$

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Fig.4(a-b-c-d-e-f) curve of Hazard function of KW-P distribution

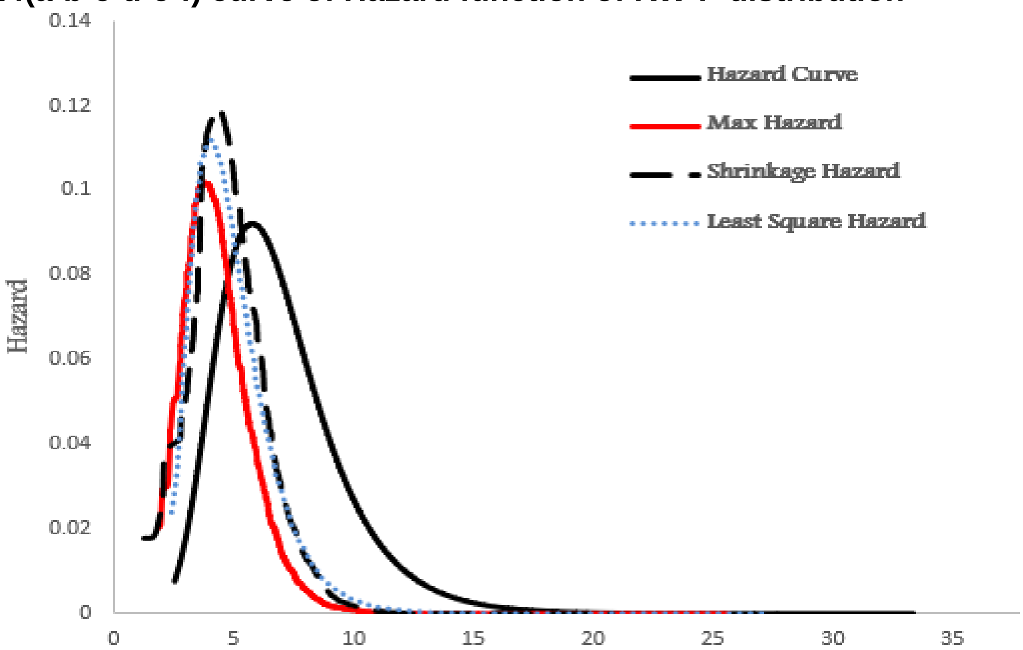

Fig.4-a Curve of Hazard function of KWP distribution with $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ and $(n = 25)$

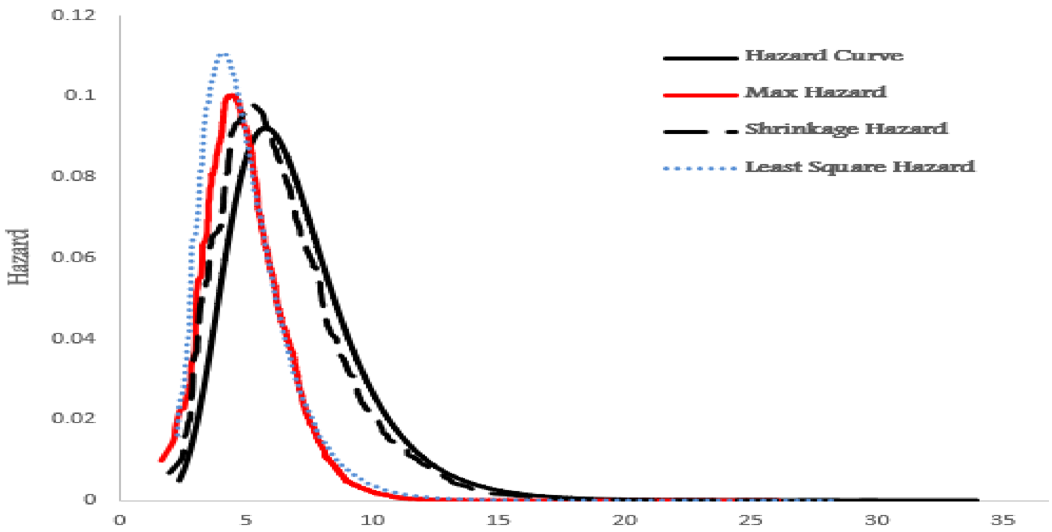

Fig.4-b Curve of Hazard function of KWP distribution with $(\alpha = 9; \boldsymbol{\beta} = \mathbf{0.2}; \boldsymbol{\theta} = \mathbf{1}; \boldsymbol{\gamma} = \mathbf{0.5})$ **and** $(\boldsymbol{n} = \mathbf{25})$

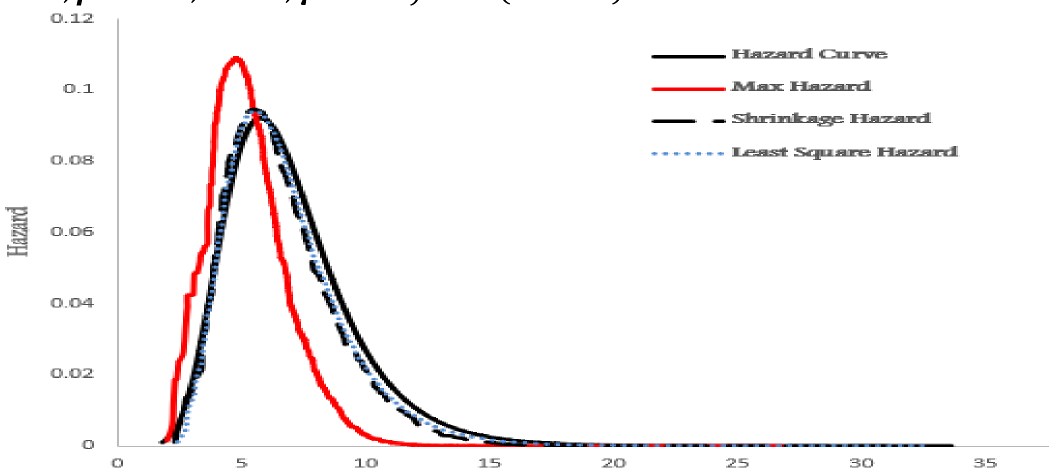

Fig.4-c Curve of Hazard function of KWP distribution with $(\alpha = 9; \boldsymbol{\beta} = \mathbf{0.2}; \boldsymbol{\theta} = \mathbf{1}; \boldsymbol{\gamma} = \mathbf{0.5})$ **and** $(\boldsymbol{n} = \mathbf{25})$

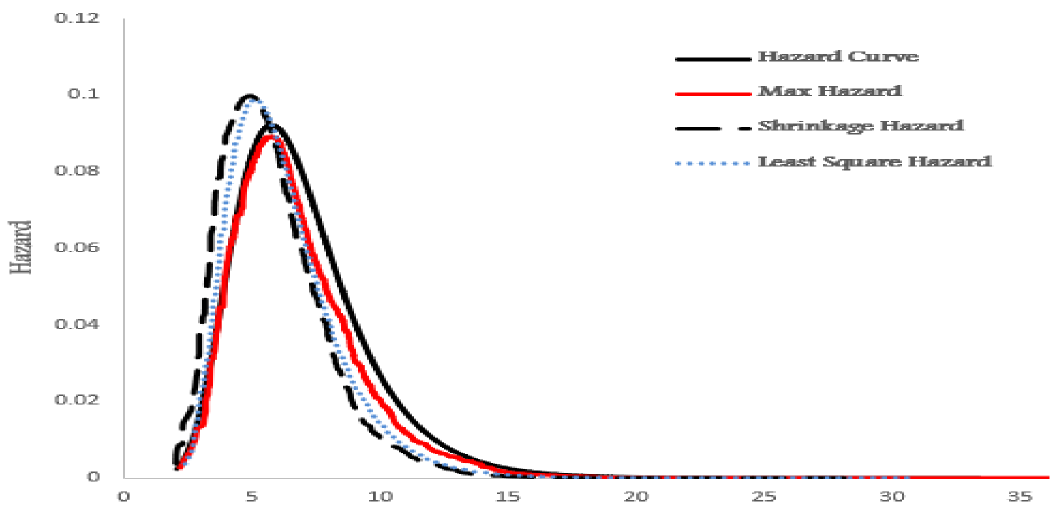

Fig.4-d Curve of Hazard function of KWP distribution with $(\alpha = 9; \boldsymbol{\beta} = \mathbf{0.2}; \boldsymbol{\theta} = \mathbf{1}; \boldsymbol{\gamma} = \mathbf{0.5})$ **and** $(\boldsymbol{n} = \mathbf{25})$

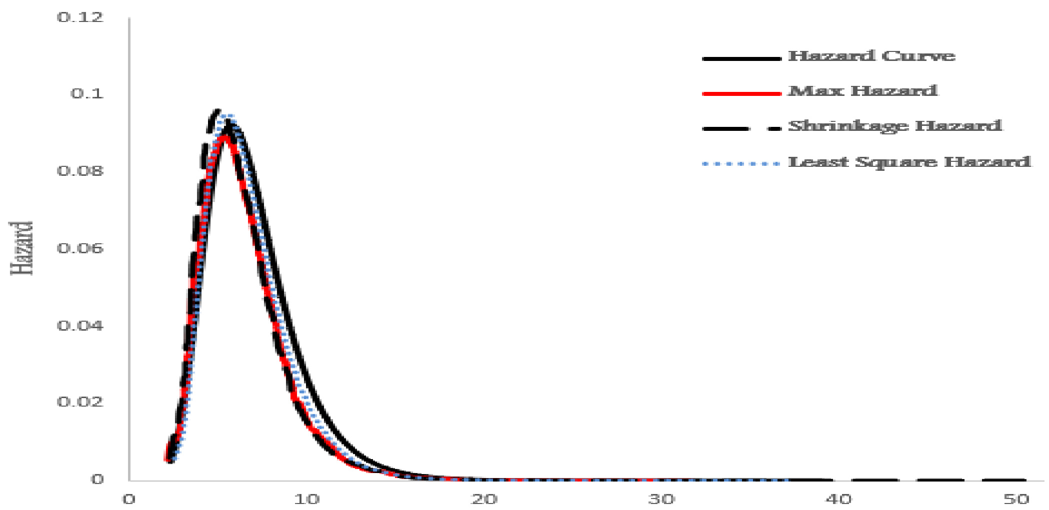

Fig.4-e Curve of Hazard function of KWP distribution with $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 25)$

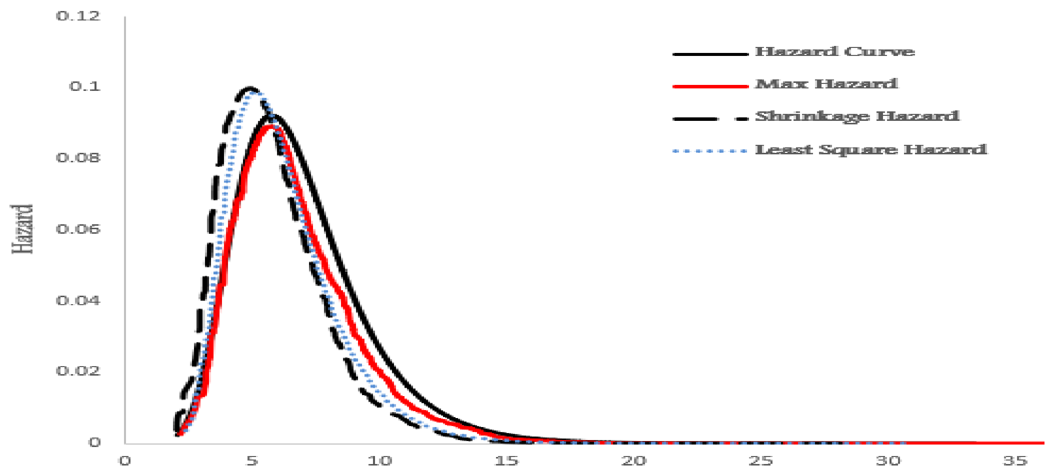

Fig.4-f Curve of Hazard function of KWP distribution with $(\alpha = 9; \beta = 0.2; \theta = 1; \gamma = 0.5)$ **and** $(n = 15)$

Divergence in curves of estimated reliability and actual reliability for small sizes of samples is clear in Fig. (3) (which represents the behavior of actual reliability and the estimated reliability by MLEM; SHRM and LSEM. While for big sizes of samples convergence appears.

Also, Divergence in curves of estimated hazard and actual hazard for small sizes of samples is clear in Fig. (4) (which represents the behavior of actual hazard and the estimated hazard by MLEM; SHRM and LSEM. While for big sizes of samples, convergence appears.

8-Conclusions:

The results of simulation tabled (2) and figs. (3);(4) show:

1- Flexibility in hazard curve of KW-P distribution.
2- Convergence in results of the MLM, LSEM and SRHM for all chosen sizes of samples.
3- accuracy in results is directly proportional to the samples' sizes.
4- Estimations of samples with large sizes have a greater accuracy in maximum likelihood method compared with SRHM and LSEM
5- Estimations of samples with small sizes have a greater accuracy in LSEM compared with MLM and SRHM

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