Quality Control Charts Using Time Series Methods

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Abstract

The control of the production process is one of the fundamental things of production companies, in order to be able to analyze their reality, and to obtain production of good quality, and with acceptable specifications, and in a way that leads to the development of the current reality of industrial companies, hence the purpose of the research came to use the methods of time series, which are dependent on the Simple Moving Average model and Simple exponential smoothing model for the purpose of creating quality control charts for moving averages, and exponentially weighted moving averages, using data from the General Battery Industry Establishment for the year 2014, it was concluded that the using an exponentially weighted moving averages chart to control any abnormal time series, is better than using the moving averages chart.

Key Word: Quality control, Simple moving average, Simple exponential smoothing,

1. Introduction

The statistical method in controlling the quality of the product is one of the important methods, for detecting the quality of a particular product, and its conformity with the specifications specified in advance by the producer. It is also one of the decision-making methods, for production managers and engineers, for the purpose of running the production process, according to the specifications set for it starting from the stage of receiving raw materials, until the final product stage, in order to obtain the required quality within the limits of the specifications specified by the manufacturer.

The statistical method used in most of our industrial facilities depends mainly, on taking random samples, from production lines, and examining and analyzing these samples, according to a distinctive property, determined by specialists in standardization and quality control. This distinctive property of production cannot always be expressed numerically or numerically, which is called the variable, such as Height, weight, size, etc. However, this distinctive property may be in the form of absolute or relative quantities such as color, design, model, etc., which are called in this trait, such an trait, is often expressed by producing units that conform to specifications, or in the industrial concept, produce defective or non-defective units.

2. Research objective

The time series link the observation value, during a specific time period, since the current information is better than its better than the previous one to influence, on the observation, this method is good in predicting and inferring the value of the observation currently being studied and in the future, because it relies on current and recent data, in the study of the observation, and from here the research objective was advantage of these methods form time series, and to integrate them into methods of quality control, in order to reach and apply some control chart as a moving averages control chart, as well as a (Exponentially Weighted moving average control chart), so these charts have the possibility of application when the number of defective units changes, as well as the Nonconforming Proportion.

3. Theoretical Side

Use some methods of time series in particularly exponential smoothing methods, in terms of theoretical side, and the relationship of these methods, to quality control charts, and the differences affecting them each.

3-1 Moving average control chart. [1], [2]

The time series model depends on a fixed process plus the error, so the model is described by the formula:

$$w_t = b + \varepsilon_t \qquad \cdots(1)$$

Whereas (ε_t) is the error value, which is distributed normally with zero mean and variance(σ_ε^2), &(b), represents the unknown parameter of the model, and if the relative importance is equal, to all observations of the phenomenon, then the parameter estimation(b), is through the method of least squares, where the sum of the squares of the error resulting from the model is reduced, as the following formula:

$$s = \sum_{t=1}^{r} (w_t - b)^2 \qquad \cdots(2)$$

Then the estimate of the moving averages, with length of the (r) cycle and the parameter (b) is described.

$$\hat{b} = \frac{1}{r}\sum_{t=1}^{r} w_t \qquad \cdots(3)$$

Where it represents the mean of the phenomenon values, and the length of (r), that which deals with all the values of the phenomenon. And that the relative importance of observations changes over time, so that the current view is more important than the previous one, so this means that in estimating (b), more important weights are given, to the observations, $w_t, w_{t-1}, \ldots\ldots, w_{t-r+1}$, and weight $1/r$, a weight equal to zero is given to the rest of the observations, $w_{t-1}, w_{t-r+1}, \ldots\ldots, w_1$, in this case, the estimate of the value of parameter b is equal to:

$$\hat{b} = (w_t + w_{t-1} + \cdots + w_{t-r+1})/r \qquad \cdots(4)$$

This is known as a simple moving average, on which the control chart for the moving averages will be based.

After explaining the theoretical side of time series, this method can be used to describe the quality control chart, to adopt a control chart on the prediction method for time series based on moving averages, with the length (r), as the main equation in calculating control limits is in the following formula:

$z_t = (w_t + w_{t-1} + \cdots + w_{t-r+1}) / r \qquad \cdots(5)$

Thus:

$ucl, lcl \Rightarrow \mu_w \mp k\,\sigma_w / sqr(r) \qquad \cdots(6)$

$ccl \Rightarrow \mu_w$

Where: w_t : Sample (t) is a measure of quality or the property.

μ_w : Average value (w).

σ_w : Standard deviation to value (w).

r : Moving average length.

3-2 Quality control chart for exponential weighted moving averages. [2], [6]

When the defective values are slightly changed, single exponential smoothing models can be used, since the model (1) remains effective in describing the phenomenon, therefore, the estimate of the parameter (b) is: $\hat{b}(t) = (1-\beta) w_t + \beta\,\hat{b}(t-1) \qquad \cdots(7)$

Assuming that $(a = 1-\beta)$, then $w_t = \hat{b}(t)$ therefore, the estimation equation for the simple exponential smoothing model is described by the following relationship:

$s_t = \alpha\ w_t + (1-\alpha)\ w_{t-1} \qquad \cdots(8)$

Whereas (α) represents the smoothing constant.

The specifications for this estimate are:

Expected of the error is: $E(s_t) = b$

Variance of error is: $var(s_t) = \frac{a}{1-a}\sigma_\varepsilon^2$

Variance of moving average is: $var(\mu_t) = \frac{\sigma_\varepsilon^2}{r}$

So, we note that the theoretical basic for this chart is based on exponential smoothing models in the time series, especially on the simple moving average model, what assumes weights between successive values of the phenomenon, these weights show the relative importance of the observation when calculating the predicted values, then calculating the limits of control, whereas, the basic model of this method is described as follows:

$z_t = \lambda\ w_t + (1-\lambda)\ z_{t-1} \qquad \cdots(9)$

The limits of the chart are described by:

$$ucl, lcl \Rightarrow \mu_w \mp k\,\sigma_w \;.\, sqr(\frac{\lambda}{2-\lambda}) \qquad \cdots(10)$$

$$ccl \Rightarrow \mu_w$$

So that:

$\mu_w = z_0$

w_t: Statistical for the sample and for the required period.

λ: Constant weight, and its value ranges between ($1 > \lambda > 0$) and there is no basis for determining the optimum value (λ).

z_t: A geometric mean moving at the period (t).

k: A constant amount between the deviation from the mean and equal to (3 or 2).

But when there is a general trend in the phenomenon that has a relationship with time, the general model is described by:

$$w_t = b_1 + b_2 t + \varepsilon_t \qquad \cdots(11)$$

That's where we expect the error($E(\varepsilon_t) = 0$), and the variance $var(\varepsilon_t) = \sigma_t^2$.

We note in the model the appearance of the linear effect, represented by the parameter b_2 in addition to the parameter(b_1), when applying the simple exponential smoothing,

$$s_t = \alpha\, w_t + (1-\alpha)\, s_{t-1} \qquad \cdots(12)$$

If the simple exponential smoothing method is matched to the outputs of equation (12), then we get:

$$s_t^{(2)} = \alpha\, s_t + (1-\alpha)\, s_{t-1}^{(2)} \qquad \cdots(13)$$

At this point, $b_2(t)$ can be estimated at the time period t through the equation:

$$\hat{b}_2(t) = \frac{\alpha}{1-\alpha}(s_t - s_t^{(2)}) \qquad \cdots(14)$$

Where, $0 < \alpha < 1$.

And that the quality control chart, based on this method is an approach to the simple exponential smoothing model in equation,(9), Knowing that the smoothing takes place in two steps, and depending on values ($s_t^{(2)}$).

4. Applied Side

4-1: Description of the data

To clarify the applied side of the two control charts shown in the theoretical side, data were used from the General Establishment for Battery Industry, Baghdad - Babel Factory, current (60) amperes, (12) volts, and the production for the year 2014. It consists of a defective rate of (150) production batches of different sizes for the purpose of ensuring that the production process for the production of liquid batteries, is under control or not, as shown in Table No. (1), noting that the sample size in the subject of the research will take the form of a non-fixed variable, and the result is that the control limits will not be fixed, where they take the wobbling shape, but the central control limit will take the fixed shape, because it represents the mean values of the phenomenon, and depending on the excel program,

Table (1)
Defective rate

1	0.05	35	0.0384615	69	0.049763	103	0.056338	137	0.0358744
2	0.047619	36	0.0471014	70	0.0363128	104	0.0398551	138	0.0520833
3	0.075	37	0.0416667	71	0.0410959	105	0.0389381	139	0.0520833
4	0.0518519	38	0.0466667	72	0.0527578	106	0.0452899	140	0.0657895
5	0.0335008	39	0.0650407	73	0.075	107	0.0771513	141	0.0530303
6	0.0419948	40	0.0709677	74	0.0566038	108	0.0583333	142	0.0551331
7	0.0416667	41	0.0486111	75	0.0574163	109	0.0461165	143	0.0645833
8	0.0505952	42	0.0462963	76	0.0486486	110	0.0442105	144	0.0566667
9	0.0454545	43	0.0545113	77	0.0612245	111	0.0333333	145	0.0452586
10	0.0365854	44	0.0729167	78	0.0650888	112	0.0583658	146	0.042517
11	0.0391389	45	0.0495495	79	0.0390625	113	0.0735294	147	0.0384615
12	0.057971	46	0.05	80	0.0719697	114	0.0598802	148	0.0625
13	0.0679612	47	0.0447761	81	0.0543478	115	0.0533333	149	0.0497738
14	0.0453515	48	0.0383838	82	0.0388889	116	0.047619	150	0.0696629
15	0.0787402	49	0.0492424	83	0.0454545	117	0.0534351		
16	0.0821918	50	0.0277296	84	0.0545977	118	0.0560538		
17	0.0392157	51	0.0458333	85	0.045	119	0.0405797		
18	0.0987654	52	0.0404762	86	0.0465116	120	0.0480769		
19	0.079646	53	0.0490196	87	0.0446927	121	0.0344203		
20	0.0423077	54	0.0458333	88	0.0725807	122	0.0436508		
21	0.0541667	55	0.0398671	89	0.0425926	123	0.0510511		
22	0.0545977	56	0.0328638	90	0.0725806	124	0.0481481		
23	0.0672043	57	0.0364964	91	0.0416667	125	0.0508482		
24	0.0722222	58	0.0636792	92	0.0478723	126	0.0666667		

25	0.0271739	59	0.0671642	93	0.0427046	127	0.0416667		
26	0.0784314	60	0.0580046	94	0.0317848	128	0.0366667		
27	0.125	61	0.0455373	95	0.0460527	129	0.0337302		
28	0.0666667	62	0.042381	96	0.0512821	130	0.025463		
29	0.0425926	63	0.0458716	97	0.0569307	131	0.0426357		
30	0.0724638	64	0.0927419	98	0.0532151	132	0.0444444		
31	0.0642857	65	0.0542636	99	0.0443459	133	0.052381		
32	0.0757576	66	0.0229226	100	0.0708333	134	0.0526316		
33	0.0535117	67	0.0551378	101	0.0354167	135	0.0677083		
34	0.047619	68	0.0489362	102	0.0532151	136	0.0569514		

4-2 Analysis of the results

4-2-1 Moving average control chart

The values of the moving average (z_t) were obtained, by applying equation no. (5), through the value of $(r = 6)$, and applying equation no. (6) To extract the control limits.

Where the standard deviation of the values of the phenomenon was extracted according to the following equation:

$$\sigma wt = \sqrt{\frac{\overline{w}}{r}}$$

Where: $\overline{w}$, Average values of the phenomenon.

r: Sample size for each batch.

The control limits for the moving average chart are shown in Figure, (1), and the value of the central limit is equal to 0.052378 , and the tables (2),(3) shown the upper and lower values.

Figure (1)

Control limits for the moving averages chart

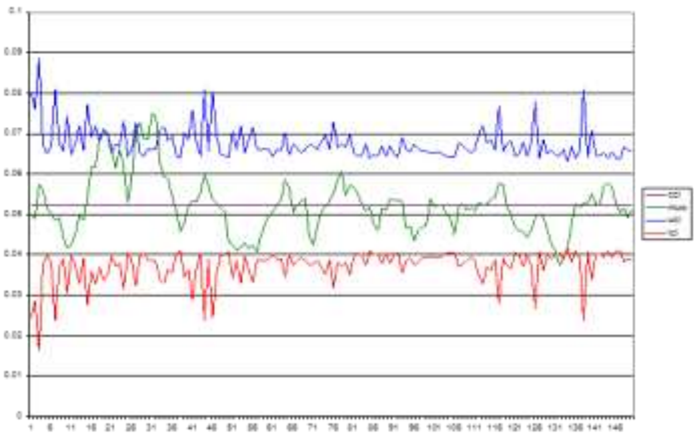

Table (2)

Upper limit for the moving averages chart

0.0804	0.0652	0.0757	0.0643	0.0654	0.0652	0.0643	0.0646
0.0761	0.0674	0.0679	0.0661	0.0644	0.0656	0.0649	0.0646
0.0886	0.0669	0.0645	0.0658	0.0646	0.065	0.0677	0.0652
0.0672	0.0733	0.081	0.0702	0.0674	0.0643	0.0644	0.0638
0.0649	0.0643	0.0657	0.0647	0.0638	0.0642	0.0677	0.0654
0.0667	0.0663	0.0804	0.0674	0.0647	0.0643	0.078	0.0639
0.081	0.0726	0.0695	0.0664	0.0645	0.0676	0.0638	0.0636
0.0677	0.0652	0.065	0.0653	0.0669	0.0672	0.0686	0.0667
0.0657	0.0644	0.0646	0.066	0.0644	0.0662	0.0649	0.0657
0.0743	0.0662	0.064	0.0672	0.0669	0.0652	0.0659	0.0657
0.0648	0.0661	0.0705	0.067	0.0649	0.0661	0.0647	
0.0675	0.0665	0.0661	0.0661	0.0642	0.0699	0.0644	
0.0719	0.0709	0.072	0.068	0.0691	0.072	0.0661	
0.0657	0.0717	0.0652	0.0696	0.0662	0.0677	0.0631	
0.0772	0.0682	0.0685	0.0661	0.0655	0.0686	0.0667	
0.0688	0.0692	0.0716	0.073	0.0673	0.0661	0.0638	
0.072	0.0646	0.0662	0.0665	0.0663	0.0769	0.0657	
0.0679	0.0638	0.066	0.0676	0.0656	0.0657	0.081	
0.071	0.0702	0.0664	0.0667	0.0656	0.0675	0.0641	
0.0698	0.0683	0.0659	0.0696	0.0652	0.0682	0.0709	

Table (3)

Lower limit for the moving averages chart

0.0243	0.0396	0.029	0.0404	0.0393	0.0396	0.0404	0.0402
0.0287	0.0374	0.0368	0.0387	0.0403	0.0392	0.0399	0.0402
0.0162	0.0378	0.0402	0.039	0.0402	0.0398	0.037	0.0396
0.0376	0.0315	0.0238	0.0346	0.0374	0.0404	0.0403	0.0409
0.0398	0.0404	0.0391	0.04	0.0409	0.0406	0.037	0.0394
0.038	0.0385	0.0243	0.0374	0.04	0.0404	0.0268	0.0408
0.0238	0.0321	0.0353	0.0383	0.0403	0.0371	0.0409	0.0412
0.0371	0.0396	0.0398	0.0394	0.0378	0.0376	0.0362	0.0381
0.039	0.0403	0.0402	0.0387	0.0403	0.0386	0.0399	0.039
0.0305	0.0386	0.0407	0.0376	0.0378	0.0395	0.0389	0.0391
0.04	0.0387	0.0343	0.0377	0.0399	0.0387	0.04	
0.0373	0.0383	0.0387	0.0387	0.0406	0.0349	0.0403	
0.0328	0.0339	0.0328	0.0367	0.0357	0.0328	0.0387	
0.039	0.033	0.0396	0.0352	0.0385	0.037	0.0417	
0.0275	0.0365	0.0362	0.0387	0.0393	0.0362	0.0381	
0.036	0.0355	0.0332	0.0318	0.0374	0.0387	0.0409	
0.0328	0.0402	0.0386	0.0382	0.0384	0.0279	0.0391	

0.0368	0.0409	0.0388	0.0371	0.0392	0.0391	0.0238	
0.0337	0.0345	0.0384	0.0381	0.0392	0.0373	0.0407	
0.035	0.0365	0.0389	0.0351	0.0396	0.0365	0.0338	

4-2-2 Quality control chart for exponential weighted moving averages

The values (s_t, s_t^2) are found by applying the equations (12 & 13), in order to use them in applying equation (9) to extracting the (Moving geometric mean values) (z_t) , at the period time(t), with assumed the$(\lambda = 0.9)$, because it given the minimum mean squared error of the values, (0.0000226) as shown in the table (4).

Table (4)

The (λ) value and MSE.

λ	MSE
0.1	0.004472
0.3	0.00501
0.5	0.0059
0.7	0.000367
0.9	0.0000226

The equation (10), was applied to extract the upper and lower limits of the control chart, where they found that the value of the central limit is equal to (0.094657439) as shown in the figure (2), and the tables (5),(6) shown the upper and lower values.

Figure (2)

Control limits Exponential weighted moving averages chart

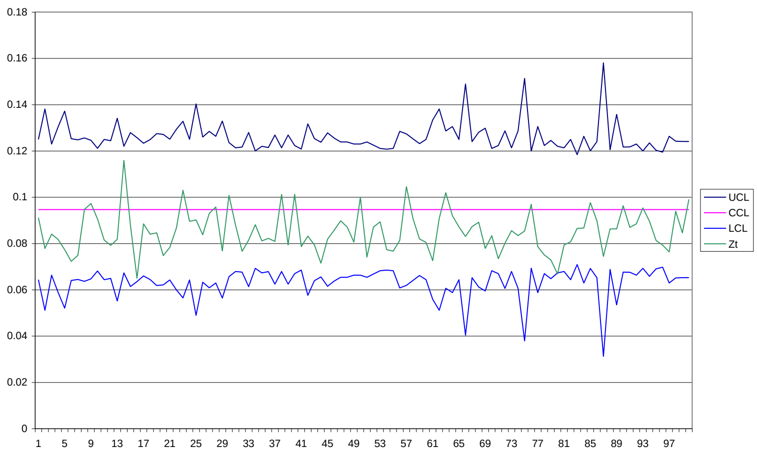

Table (5)

Upper limit of the Exponential weighted moving averages chart

0.125	0.1251	0.1208	0.1334	0.1214
0.1381	0.1294	0.1317	0.1381	0.125
0.123	0.1328	0.1254	0.1286	0.1184
0.1305	0.125	0.1237	0.1305	0.1263
0.1372	0.1403	0.1278	0.125	0.1201
0.1253	0.126	0.1256	0.1489	0.1241
0.1248	0.1284	0.1239	0.1241	0.158
0.1256	0.1263	0.1239	0.1281	0.1205
0.1246	0.1329	0.123	0.1298	0.1358
0.1212	0.1236	0.123	0.1211	0.1217
0.125	0.1214	0.1239	0.1223	0.1217
0.1244	0.1217	0.1225	0.1287	0.123
0.1341	0.1279	0.1211	0.1214	0.12
0.122	0.12	0.1208	0.1286	0.1235
0.1279	0.122	0.1211	0.1514	0.1203
0.1257	0.1215	0.1285	0.12	0.1195
0.1233	0.1269	0.1274	0.1305	0.1263
0.1249	0.1214	0.1253	0.1223	0.1242
0.1275	0.1269	0.1232	0.1245	0.1241
0.1272	0.1223	0.125	0.122	0.1241

Table (6)

Lower limit of the Exponential weighted moving averages chart

0.0644	0.0642	0.0685	0.0559	0.0679
0.0512	0.0599	0.0576	0.0512	0.0644
0.0663	0.0565	0.0639	0.0607	0.0709
0.0589	0.0643	0.0656	0.0588	0.063
0.0521	0.049	0.0615	0.0644	0.0692
0.064	0.0633	0.0638	0.0404	0.0653
0.0645	0.0609	0.0654	0.0653	0.0313
0.0637	0.063	0.0654	0.0612	0.0688
0.0647	0.0564	0.0663	0.0595	0.0535
0.0682	0.0657	0.0663	0.0682	0.0676
0.0644	0.0679	0.0654	0.067	0.0676
0.0649	0.0676	0.0668	0.0606	0.0663
0.0552	0.0614	0.0682	0.0679	0.0693
0.0673	0.0693	0.0685	0.0607	0.0658
0.0614	0.0673	0.0682	0.038	0.069
0.0636	0.0679	0.0608	0.0693	0.0698
0.066	0.0625	0.0619	0.0588	0.063

0.0644	0.0679	0.0641	0.067	0.0651
0.0618	0.0625	0.0662	0.0648	0.0652
0.0622	0.067	0.0644	0.0673	0.0652

5. Conclusions and recommendations

1. From observing the moving averages chart depending on the simple moving average, and as shown in Figure (1), we find that there are values outside the limits of control, from batch (26) to batch (34), meaning that there are outliers, and this indicates that the presence of impurities outside the limits of statistical control, there is an indication that there are non-random factors that affect this characteristic and must be identified with the necessary measure to prevent recurrence.
2. We have noticed the exponential weighted moving averages chart, and after applying exponential smoothing on the data, as in Figure (2), we find that all values fall within the limits of statistical control, and this indicates that they can be used to control the quality of products in the future.
3. Using an exponentially weighted moving averages chart to control any abnormal time series, is better than using the moving averages chart.
4. Time series methods are the basis for creating quality control charts of product quality.
5. The necessity to focus on the quality of the raw materials in order to reduce the impurities as much as possible because they are reflected in the quality of production.
6. Quality improvement through the use of statistical methods, including quality control, which helps in determining the progress of the production process in order to improve quality at the lowest possible cost.

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