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Using the Heston model in pricing European Options "Research of PhD dissertation"

Kafaa A. Essaa Alborgeef * P.D. Haider N. Gali Al-Furaigy **

Abstract:

Options pricel are generally determined by a tradeoff between risk and return. Options - prices models generally rely on the volatility index in stock market prices, which represents the risk to the buyer or the option seller in the financial market. This has led to the emergence of many models of pricing options that generated different values for one option according to the assumptions, parameters and techniques used in each model , and hers lies the problem of this research, that the application of pricing models that assume volatility instability can produce option's Prices closer to market prices. The aim of this research is to apply one of the models of random volatility in the samples-prile of options, namely Heston Model. To achieve the objective of the research, the historical published prices of the shares of (4) large companies operating in the US technology sector for the period from (29/12/2015) to (19/2/2016) for a sample consisting of (12) options on the shares of those companies. The research had a set of conclusions and recommendations, the most important of which is that the prices of Heston model almost close to its market price, but this model depends on the accuracy of its prices on the change in the value of the parameters it consists of as well as the complexity that accompanies its application.

المستخلص:

تتحدد أسعار الخيارات بشكل عام وفقاً للمبادلة بين العائد والمخاطرة، إذ تعتمد نماذج تسعير الخيارات عموماً على معلمة التقلب في أسعار الأسهم في السوق المالي، والتي تمثل المخاطرة التي يتعرض لها مشتري أو محرر الخيار في السوق المالي . لذا ، فقد شغل هذا الأمر حيزاً واسعاً من

* باحثة .
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الجدل المعرفي والفكري خاصةً بعد تطور الأسواق المالية وزيادة الإستثمارات فيها ، الأمر الذي أدى الى ظهور نماذج كثيرة لتسعير الخيارات التي ولّدت قيماً مختلفة للخيار الواحد على وفق الإفتراضات والمعلمات والتقنيات المستخدمة في كل أنموذج ، وهنا تكمن مشكلة البحث والتي تمثلت في أن تطبيق نماذج تسعير الخيارات التي تفترض عدم ثبات التقلب بإمكانها إنتاج أسعار خيارات قريبة من أسعار السوق . يهدف هذا البحث الى تطبيق أحد نماذج التقلب العشوائي في تسعير الخيارات وهو أنموذج (Heston) ، ولتحقيق هدف البحث فقد تم إعتماد الأسعار التاريخية المنشورة لأسهم (4) شركات كبيرة تعمل في القطاع التكنولوجي الأمريكي للمدة من (29/12/2015) لغاية (19/2/2016) لـعينة تتألف من (48) خياراً من الخيارات المبرمة على أسهم تلك الشركات. وتوصل البحث الى مجموعة من الإستنتاجات والتوصيات والتي من أهمها أن أنموذج (Heston) يقوم بتسعير جميع الخيارات تقريباً بشكل قريب من سعرها السوقي ولكن هذا الأنموذج يعتمد في دقة أسعاره على التغير في قيمة المعلمات التي يتكون منها فضلاً عن التعقيد الذي يرافق تطبيقه .

The first topic
Research Methodology

1-1 Research problem:

The pricing of options is one of the most important topics in the field of financial management, which has taken a large place in the literature of modern financial thought, where it generated a great philosophical debate on the selection of models for pricing options, especially after the development of financial markets and increased investments, which led to the multiplicity of these models and Different values were generated for one option according to the assumptions, parameters and techniques used in each model. This is related to volatility, which is the basis of pricing options, in turn influenced the selection process by the investor. This shows the first part of the research problem and related to the cognitive side.

The other side of the research problem is related to its practical dimension: "The prices of the Call options resulting from the Heston random Volatility model would be close to market prices and will therefore affect the investor's behavior in making decisions about the option centers according to his preferences for return and risk, In trading options markets. "

1.2 The importance of research:

The importance of this research is highlighted by addressing one of the important areas of contemporary financial theory related to the quantitative aspect of pricing options, in order to give a clear idea of how to determine the market prices of the European Call options.

The importance of research is represented by adopting a framework that contributes to enhancing the knowledge and practical aspects of the pricing of options, and the determination of their fair theoretical

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value through the application of a model is an important and fairly modern models in the pricing of options, which is the Heston model, This is the case with the increasing importance of financial markets on the one hand, and the increasing risks associated with them on other.

Finally, this research contributes to addressing a subject that suffers from relative scarcity in Arab libraries, and a clear lack of theoretical and practical framework describing the basic dimensions of option contracts, and trying to clarify the important bases in pricing options.

1.3 Research Objectives:

The objectives of the research are to :

1. Explain the technique of the Heston pricing model in the financial markets, and present and test how this model which is one of the most important pricing models of European financial Call options, can be applied, with the use of mathematical and statistical techniques in its calculation.
2. Determine whether the change in the value of the parameters on which the model is based will lead to a difference in prices resulting from the Heston model and its proximity to market prices, and hence the different prices of options to the financial market.

1.4 Research Hypothesis:

This research has based on the following main hypothesis: "The change in the initial values of the parameters on which the Heston model is based, leads to a change in the prices of European Call options, leading to a difference in investment decision making."

1.5 Research Sample and Duration:

For the purpose of achieving the objectives of this research, the research sample consists of (12) Call options on the shares of four giant American companies operating in the US technology sector. These options have been divided according to strike prices as shown in the following table:

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Table (1)
Sample study companies

Number of Options	Company Code	Company Name	
3	DATA	DATA TABLEAU	1
3	YHOO	YAHOO!	2
3	IBM	International Business Machine	3
3	MSFT	Microsoft	4
12		Total options	

Source: Prepared by the researcher

These options have been written on 29/12/2015 with a maturity date of 3 days. All information related to the options has been obtained from the Yahoo Finance website and CSV (Comma separated values) .

The selection of this sample of companies and options was in accordance with the following criteria:

1. It is one of the most companies to write options on their shares.
2. Provide adequate data during the study period.
3. There is no an interruption in the dealings on stock options during the study period.

The non-linear quadratic function (Isqnonlin) has been used by Appling the MATLAB program to obtain model prices. However, when starting the application of the program, it is necessary to estimate the initial values of these parameters. In this research We have were based on the study of (Ye, 2013) on the estimation of these five parameters, and then change the value of each of these parameters and study the effect on prices of options through the use of six (Heston A), (Heston B), (Heston C), (Heston D), (Heston E), and (Heston F). The researchers have introduced different values for the parameters And then get the Call option prices for this model. The different initial values of the five parameters selected can randomly be shown by researchers in Table 2 :

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Table (2)
The various initial values of the five parameters according to the visions of researchers

Researcher different random values of the five parameters					Researcher
θ	ρ	V_0	σ	k	
0.46	- 0.64	0.05	2.78	0.75	(Ye, 2013)
0.92	0	0.2	0.5	0.02	
0.05	- 0.5	0.07	0.46	2	(Yang, 2013)
0.16	- 0.8	0.16	0.1	10	(Karlson, 2011)
0.2	0	0.2	0.6	0.1	(Poon, 2011)
0.05	- 0.5	0.15	0.5	3	(Moodly,2005)
0.057	- 0.75	0.16	0.7	5	

Source: Prepared by the researcher

In this research, the initial values adopted by Ye, 2013 and the two cases mentioned in the table above have been used. The prices of the Call options for the companies are calculated according to Heston and the Time to maturity used in this research.

The Second Topic
Theoretical Part of research

The option is a Contract between two parties, one of which guarantees the other party the right to buy or sell a financial instrument under certain conditions agreed yon between the parties. The buyer party will pay a premium to the selling party to be able to buy or sell the financial instrument without obligation to do so. (Walmsley, 2000: 285) This premium reflects the price of this option. Volatility in asset returns is one of the most important concepts in pricing options, where it can be considered an indicator of uncertainty related to the price of an asset. It also plays an important role in many financial applications and its main use is to estimate the value of risk. It is the main parameter when pricing derivatives generally and options In particular, all modern pricing techniques depend on the volatility parameter. Volatility is also used to assess risk management and portfolio management in general (Ladokhin, 2009:1). The main approaches to pricing options based on volatility types will be reviewed.

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2.1 Options Pricing Approaches

The problem of pricing options in a manner consistent with the volatility smile has attracted considerable attention in the literature of financial management, and many suggested approaches to pricing options have emerged. The main approaches to pricing can be distinguished hasing on the type of volatility as follows: (Ruf, 2012: 17) (Mitra, 2009: 17-18)

- Constant Volatility Models
- Time Dependent Volatility models
- Time-based volatility models and the underlying asset price, called Local Volatility models
- Stochastic Volatility models

It is important to note that there is no model for pricing options that is fully acceptable, because each market has its own preferences on these models (Kangro, 2003: 4). In this Research, the Heston model will be used as an example of Stochastic Volatility models.

2.1.1 Heston Model

Steven Heston, a professor at Yale University, suggested a Stochastic Volatility pricing model, later known as the Heston model, in his 1993 paper "A Closed- Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options"(Heston,1993: 1). This model has become the most widely used in pricing options, offering a closed-Form solution to pricing European Call options and its ability to produce a Volatility smile that can be obtained from market data.

A) The Model's Equation

Heston, according to his 1993 model, assumes that the Stochastic Volatility model consisted of two processes known as Brownian motion or the Weiner process: one for underlying price and one for price volatility. Heston assumed that the price of shares S_t follows the log-normal distribution, The volatility of the stock price of V_t follows the Mean Reversion process or the process of CIR, so the model is determined by the following relationships: (Yang, 2013: 25) (Gatherl & Lynch, 2002: 4) (Gong et al., 2014: 2)

$$\mathrm{dS_t} = \mu \mathrm{S_t dt} + \sqrt{\mathrm{V_t}}\mathrm{S_t dW_t^s} \ldots\ldots\ldots\ldots\ldots (1)$$

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$dV_t = K(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^v \ldots. \ldots \ldots (2)$

$dW_t^s dW_t^v = \rho dt \ldots \ldots \ldots. (3)$

The parameters of the above model can be illustrated as follows:

μ: represents the expected rate of return per share.

θ: Long-Term Mean Variance in the share price.

K: Mean Reversion for Long-Term Variance in the share price.

σ: represents Volatility of Volatility.

S_t: represents the share price in time t.

$\sqrt{V_t}$: represents the volatility of the share price in time t

dW_t^s: represents The Brownian motion of the price of the stock

dW_t^v: represents The Brownian motion of the variance of the stock price

ρ: correlation coefficient between dW_t^s and dW_t^v.

The first equation of the model is similar to the Black and Scholes model, but the main contributory assumption is that the volatility is Stochastic and not constant. This is illustrated by the addition of another random process that ensures this Stochastic Volatility, The Stochastic process is dW_t^v. The Heston model meets the B&Sch model when V_t is constant. The most important part of the model is the second equation, which consists of two parts: (Nguyen & Solvang, 2013: 31) (Helgadottir & Ionescu, 2016: 23)

The first part is that the variability is characterized by Mean Revertion (Cox, Ingersoll & Ross 1985), which is CIR. This process can be explained by the fact that the current variance and its symbol V_t meet at some point with long-term volatility (θ). Whether the current variation is high or low today, it must have some dynamics that carries it to the long-term medium of this variation. The Kappa (K) symbol denotes the Mean Reversion and determines how quickly this convergence between the current variance and the long-term average level is .

The second part of the equation expresses the variance Volatility parameter (σ), which denotes the magnitude of the random shock of this Volatility, multiplied by another random process (dW_t^v) in a way that allows the model to be randomized. The correlation between the two random processes dW_t^s and $dW_t^v$is represented by the ρ parameter, which represents the correlation coefficient. The second equation of the model ensures that the variance cannot be negative to the positive values of k and, θ, but if the variance is close to zero, this equation ensures Return of variance to the long-term average.

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The price of the European option can be calculated according to the Heston model using the following formula: (Nykvist, 2009: 13), (Gong etal, 2014: 3), (Poon, 2011: 7), (Moodley, 2005: 10)

$C(S, v, t) = SP_1 - KP(t, T)P_2$............................(4)

The above equation consists of two parts: The first part represents the present value of the spot price of the underlying asset at maturity, while the second part of the equation is the present value of strike price (Ye, 2013: 25)

The Heston model includes five parameters on which the model is based when used to calculate the Call option price. These five parameters are (k, ρ, σ, v, θ) The impact of each parameter can be summarized as follows: (Bauer, 2012: 35-36) (Gauthier et al, 2009: 8-9) (Moodley, 2005: 2-7)

1. Correlation parameter (ρ):

This parameter indicates the correlation between the volatility of the underlying asset price and its logarithmic returns, which affects the widening of the distribution tails of these returns. If ρ> 0, the volatility will increase as long as the price / value of the asset increases, which will expand and widen the right tail and shrink and compress the left tail of the distribution, resulting in a distribution with a thick right tail. The greater the volatility, the greater the risk to the writer of the Call option, which leads to a higher price demand from the buyer of the option in exchange for it.

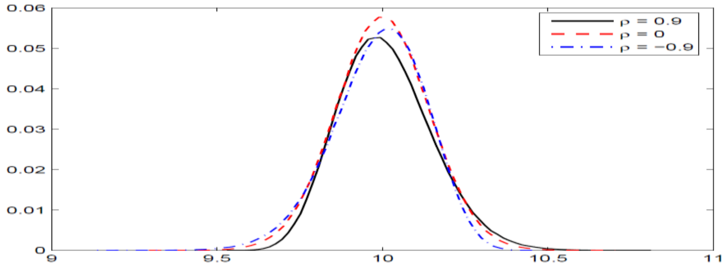

Figure (1)

The Effect of ρ on the deviation of the density function

Source: Moodley, Nimalin, "The Heston Model: A Practical Approach with MATLAB Code", An Honours Project Submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, South Africa, in Partial Fulfillment of the requirements of the Bachelor in Advanced Mathematics of Finance, 2005.

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Conversely, if $\rho < 0$, volatility will decrease when the return / price of the asset decreases, thus expanding and widening the left tail and shrinking and compressing the right tail of the distributions, resulting in a thicker tail. As a result of the low volatility, the option price will decrease as a result of the lower risk to the writer of this option.

2. Variability of Variance σ

This parameter affects flattening at the Peaks of distributions. When the value of volatility is zero, the variance is zero and the logarithmic returns will be distributed naturally. When there is an increase in the value of the volatility, the flattening will increase, creating thick tails in both directions. The effect of variability on flattening can be shown in the probability density function in Fig (2).

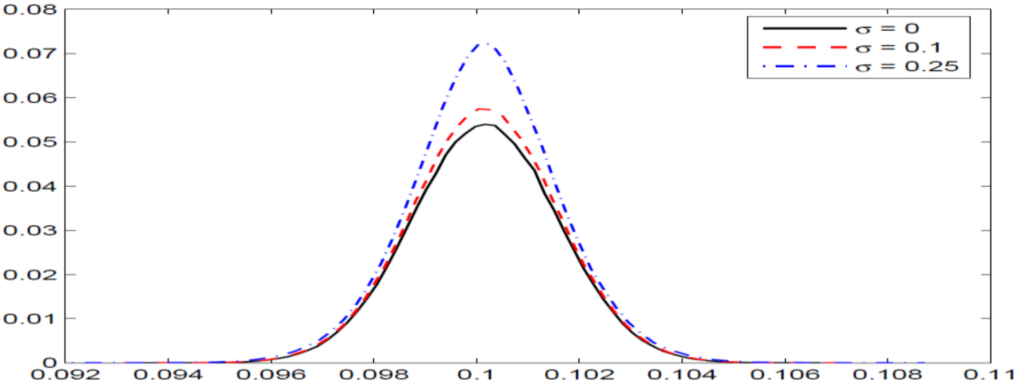

Figure (2)

The Effect of change in σ on flattening in probability density function

Source: Moodley, Nimalin, "The Heston Model: A Practical Approach with MATLAB Code", An Honours Project Submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, South Africa, in Partial Fulfillment of the requirements of the Bachelor in Advanced Mathematics of Finance, 2005.

Again, the effect of change in flattening in distributions affects implied volatility. Figure (3) shows how volatility affects the implied volatility smile. The higher the value of the volatility, the clearer the smile. The high value of the volatility means that the volatility is more volatile. This means that the market has a greater chance of extreme price movements. As a result, the Puts options writers should ask for higher fees from the Call options writers Who should ask for a lower fee for the same strike price.

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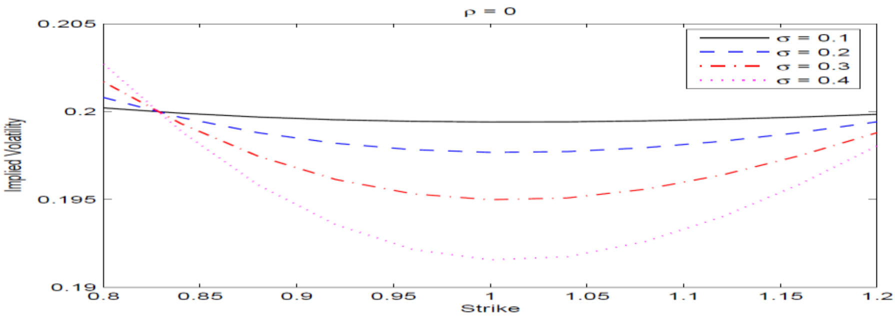

Figure (3)

The Effect of σ on the implied volatility smile

Source: Moodley, Nimalin, "The Heston Model: A Practical Approach with MATLAB Code", An Honours Project Submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, South Africa, in Partial Fulfillment of the requirements of the Bachelor in Advanced Mathematics of Finance, 2005.

3. Initial Variance parameter v_0

The change in the initial variance v_0 modifies the height of the implied volatility curve rather than the change in the shape of the curve. The increase in the initial volatility level v_0 moves the smile upward, meaning that the more fundamental volatility for each range of strike prices, the greater the risk that option writers will request additional premium from option buyers. At the same time, Buyers of these options will expect higher returns when executed. Figure (4) illustrates this behavior.

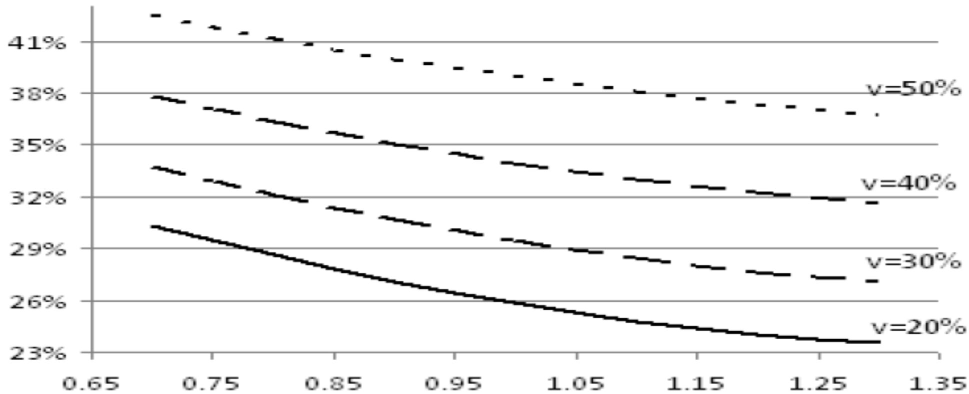

Figure (4)

The Effect of change in v_0

Source: Gauthier, Pierre & Rivaille, Pierre – Yves H., "Fitting the Smile – Smart Parameters for SABR and Heston", 2009.

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4- Long Run Variance parameter θ

The effect of both the long-term variance θ and the initial variation v_0 is similar to the implied volatility smile. The higher the variance, the higher the implied volatility curve will be, without changing the shape of the smile. This means that the more volatility in stock prices for each range of strike prices, the greater the risk that Leading to option holders seeking additional premium from option buyers, while at the same time buyers would expect higher returns when executed. Figure (5) shows the effect of long-term variability on the implied volatility smile.

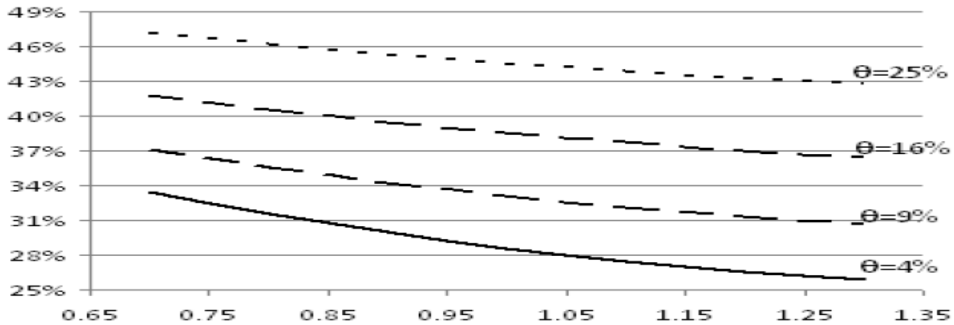

Figure (5)

The Effect of change in θ on Volatility Smile

Source: Gauthier, Pierre & Rivaille, Pierre – Yves H., "Fitting the Smile – Smart Parameters for SABR and Heston", 2009.

5. Mean Reversion parameter k

The Mean Reversion can be interpreted as a representation of the degree of clustering of volatility. Volatility Clustering can be observed in the market and means that large price movements are followed by large price movements, while small price movements are likely to be followed by small price movements. The Mean Reversion parameter k controls the curve of the Volatility smile curve, where the increase in the Mean Reversion parameter increases the flattening of the implied volatility smile. It should be noted that the decline in the mean Reversion parameter has the same effect as the increase in variance of Volatility in terms of curve bending. It can be concluded that by increasing the average rebound, this will result in lower implied volatility of the various execution rates and thus lower risk to both the buyer and option editor, which will result in a reduction in the premium requested by the option

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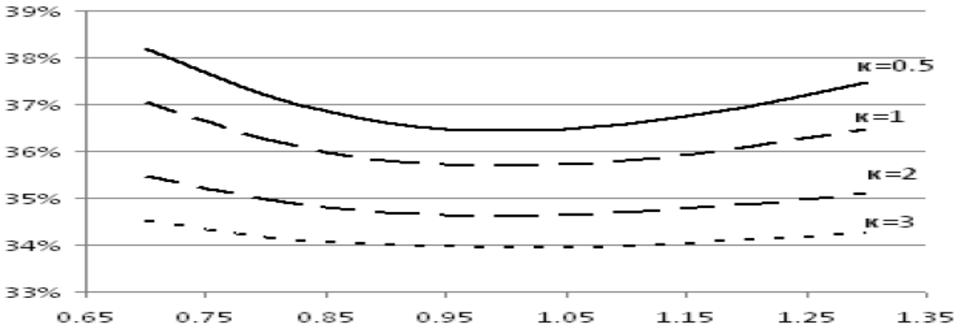
editor from a buyer. Figure (6) shows the effect of the change in the Mean Reversion parameter on the implied volatility smile.



الشكل (6)

The Effect of change in k parameter on Volatility Smile

Source: Gauthier, Pierre & Rivaille, Pierre – Yves H., "Fitting the Smile – Smart Parameters for SABR and Heston", 2009.

The characteristics of the Heston model with its five parameters enable this model to produce a very large number of distributions, which makes the model very flexible and robust and thus can address the shortcomings of the B&Sch model. It provides a pricing framework for a variety of options Closer to reality. Because of the difficulty of calculating the parameters involved in the model and its complexity, the researchers have followed the estimation of these parameters through the knowledge of the calibration of the model (Heston).

C) Modeling Calibration

A Good calibration is a legitimate process and it is necessary to obtain accurate results for a complex model such as the Heston model (Nguyen, 2013: 52). As mentioned above, the Heston model consists of five parameters whose values need to be estimated, leading to the complexity that accompanies the Stochastic Volatility models in general. The most common approach used to estimate the values of these parameters is to reduce the error or difference between model and market prices, usually expressed by the problem of non-linear Least -Squares Optimization Problem (lsqnonlin), which results in market prices Valid for options. (Moodley, 2005: 21), (Crisostomo, 2014: 13). i.e:

$$\min S(\Omega) = \min \sum_{i=1}^{N} w_i \left[C_i^{\Omega}(K_i, T_i) - C_i^{M}(K_i, T_i)\right]^2 \ldots\ldots\ldots\ldots (5)$$

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whereas:

(Ω) :A vector represents parameter values, i.e.: $\Omega = \{v_0, k, \sigma, \theta, \rho\}$

$C_i^{\Omega}(K_i, T_i)$: Options prices produced by the model

$C_i^{M}(K_i, T_i)$: Prices of options published in the market

K_i : Strike prices for options

T_i : Time to maturity for Options

N : Number of options used in the calibration process

w_i: Weights Which obtained through the relationship $w = \frac{1}{|bid_i - ask_i|}$

In a manner that achieves the following relationship: (Moodley,2005:22)

$$\min S(\Omega) = \min \sum_{i=1}^{N} w_i \left[C_i^{\Omega}(K_i, T_i) - C_i^{M}(K_i, T_i)\right]^2 \leq \sum_{i=0}^{N} w_i [bid_i - ask_i]^2 \dots (6)$$

This means that the model is not required to produce prices that exactly match the average market price, but is averaged within the bid-ask margin. (Moodley, 2005: 24)

In this research, MATLB'S lsqnonlin software has been used to calibrate the Heston model. It is worth mentioning that the calibration process requires appropriate initial values for these parameters. In this research, the initial values used in the study (Ye, 2013: 32) have been applied. Six cases of different values of these parameters were applied and the prices resulting from the model have been examined under each change Parameter of these parameters. Heston A prices have been the base case, and each of the five parameters is changed and model prices have been tested.

D. Advantages of the model

The Heston model has the following features:(Yang,2013:31), (Homescu, 2014: 5), (Mital, 2015: 1)

1. It is offering a closed-form solution to pricing the European Call option, that is, there is a direct equation in the model through which the option price can be obtained arithmetic directly, and not in terms of another variable.
2. Being able to explain the characteristics of the share price when it has a natural lognormal and Gaussian distribution.
3. It is matching the surface of implied volatility of option prices in the market and therefore has a more realistic behavior.
4. Model prices tend to be more in line with market movements.

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E. Criticisms of the model

Although the Heston model is widely used in financial research, it still has some drawbacks, including: (Yang, 2013:31), (Helgadottir & Ionescu, 2016: 37), (Homescu, 2014: 5), (Orosi, 2010: 49-50)

1. It is difficult to calculate and estimate the appropriate values of parameters for random sample testing.
2. The prices produced by the model are sensitive to parameters, so the efficiency of the model is based on testing, calibration and evaluation.
3. The model cannot accurately price options with long-term maturities.
4. The model is calibrated and tested by optimizing the lower squares, requiring special attention to ensure the stability of the parameters.
5. Any change in any of the Mean Reverting Parameter or Volatility of Volatility Parameter requires a re-calibration of the other parameters.
6. Model prices do not usually match well with Skew's and Smile of Market in the short term.

The Third Topic
Practical Part of Research
Financial Analysis of European Call Options Prices Resulting from the Heston Model

3.1 Option price analysis with (3) days to Maturity:

Table (3) refers to the prices of the European Call options on the shares of the four companies. The sample of the research is (12) options. These options have a duration of (3) days to maturity and were written on (29/12/2015) and expired at (31/12/2015).

The table shows that the average spot price of the sample shares of the study sample (81.365) dollars. IBM shares achieved the highest spot price of (139.78) dollars, while the lowest share price of YHOO Company which reached (34.04) dollars. The table shows that the average strike price is(75.9166) dollars. The losing option on IBM shares has the highest strike price of (142) dollars, while the winning option on YHOO shares has the lowest strike price of (25) dollars.

Table (3) also shows the values of the five parameters that affect the prices of the Heston model. In (Heston A), which is the Base Case in this analysis, the average price was (6.6142)$, which is lower than the average market price of (6.65)$. This is reflected the match between the price of the model and the market price for this period. The absolute value of the difference between (Heston A) and market prices was (0.10643)$. The highest difference was (0.6897)$ to the Options on the

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shares of Data, while the Options on MSFT shares has the lowest difference about (0.0097) $.

The average price of (Heston B) is (6.62669)$ Which is Closer to the average market price for this period. The absolute value of the difference between the (Heston B) and market prices was (0.110225)$, the highest difference is (0.6683)$ which is to the equal option on the shares of Data, and was shared with the price of (Heston A), While Options on Shares of the (IBM) company achieved the lowest difference about (0.0062) $. When comparing the prices of this column with the average price of the (Heston A), which represents the base case shows the increasing in the average price of the column (Heston B). This is due to the increase in the value of (k) parameter from (0.75) to (2), which represents an increase in the speed of volatility to return to the average level with the stability of other parameter values.

For the (Heston C) column, the average price was (6.62273) $,and when compared to the average market price, the average (Heston C) prices were lower than the average market prices. the average absolute values of the difference between the prices of the column (Heston C) and market prices is equal to (0.1086) $, and the highest difference was (0.6754) $ of the equal option on stock of the company (DATA). this shares with the prices of (Heston A)& (Heston B) .while the loser option on the stock of (IBM) achieved the lowest difference of (0.0013)$ and this shares with the prices of (Heston B). When comparing the prices of this column with the average price of the column (Heston A), which represents the base case, has been found that the average price of the (Heston C) is higher than the average market price because of increasing the value of the long-term variance parameter (ϴ) from (0.46) to (0.92) .

The average price of the (Heston D) column was (6.61873)$. When calculating the average absolute values of the difference between the prices of the column (Heston D) and market prices, it was found that equal to (0.1036)$, and the highest difference equal to (0.6936)$ which has been achieved by the equal option to the stock of the (DATA) company and this shares with (Heston A) (Heston B) and (Heston C). while the winning option on the stocks of (MSFT) company achieved the lowest difference of (0.0097) $, and this shares with the prices of (Heston A). When comparing the prices of this column with the average price of the column (Heston A), which represents the base case, the average price of this column is shown to match the base condition

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because the value of the correlation parameter (ρ) is changed from (-0.64) to (0) and it will not appear in the price of the model, because it is related to the change in the value of one stock with the change in the average volatility of this stock.

The average price of Heston E is (6.626325) $ which was lower than the average market price. When calculating the average absolute values of the difference between the prices of the column (Heston B) and market prices appeared to be equal to (0.112708) $, and the highest difference (0.669)$ of the equal option on the stocks of the (DATA) company, and this shares with (Heston A),(Heston B), (Heston C) and (Heston D), while the Option on the stock of MSFT company achieved the lowest difference of (0.0105)$. When comparing the prices of this column with the average price of the column (Heston A), the average price of this column is higher than the base state due to the increase of the value of the parameter V_0 from (0.05) to (0.2).

As for Heston F, the average price was (6.330875)$, which is much lower than the average market price. When calculating the average absolute values of the difference between (Heston F) prices and market prices, it is worth (0.319658) $, the highest difference was (0.8983)$ of the Option on the shares of the (DATA) company .while the loser option on the stocks of (YHOO) company achieved the lower difference (0.0032) $. When comparing the prices of this column with the average price of the column (Heston A), the average price of this column is lower than the base state due to the low value of the volatility index (σ) from (2.78) to (0.5) . In order to sequence the average absolute values of the difference between the market price and the prices of the six Heston model from the lowest to the highest , the Heston D prices, which are equal to (0.1036)$, are the closest to market prices. The (Heston F) prices are the most distant Which equals (0.3196) $.

At the end of this analysis we can say that we have proved our Hypothesis "The change in the initial values of the parameters on which the Heston model is based leads to a change in the prices of European Call options, leading to a difference in investment decision making,". The change in the prices of the European Call options on the stocks of the sample companies is observed as a result of the change in the initial values of the five parameters on which the Heston model is based.

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Table (3)

the prices of the European Call options on the shares of the four companies Resulting from the Heston Model

k= 0.75	Ɵ= 0.46	σ= 2.78	ρ=-0.64	V_0= 0.05	Heston A
k= 2	Ɵ= 0.46	σ= 2.78	ρ=-0.64	V_0= 0.05	Heston B
k= 0.75	Ɵ= 0.92	σ= 2.78	ρ=-0.64	V_0= 0.05	Heston C
k= 0.75	Ɵ= 0.46	σ= 2.78	ρ= 0	V_0= 0.05	Heston D
k= 0.75	Ɵ= 0.46	σ= 2.78	ρ=-0.64	V_0= 0.2	Heston E
k= 0.75	Ɵ= 0.46	σ= 0.5	ρ=-0.64	V_0= 0.05	Heston F

Days to Expiration : 3 Days
Time to Maturity : 0.0082191
Interest Rates : 0.0001

Company	Spot Price	Strike Price	Market Price	Heston A	$\lvert Market - Heston\ A\rvert$	Heston B	$\lvert Market - Heston\ B\rvert$	Heston C	$\lvert Market - Heston\ C\rvert$	Heston D	$\lvert Market - Heston\ D\rvert$	Heston E	$\lvert Market - Heston\ E\rvert$	Heston F	$\lvert Market - Heston\ F\rvert$
	56.55	44	12.525	12.5347	0.0097	12.5777	0.0527	12.5667	0.0417	12.5153	0.0097	12.5512	0.0262	12.0102	0.5148
MSFT	56.55	56	0.705	0.8023	0.0973	0.8114	0.1064	0.8085	0.1035	0.7538	0.0488	0.811	0.106	0.6854	0.0196
	56.55	60	0.16	0.149	0.011	0.1499	0.0101	0.1495	0.0105	0.1393	0.0207	0.1495	0.0105	0.1492	0.0108
	95.09	70	24.9	25.017	0.117	25.0217	0.1217	25.02	0.12	25.0508	0.1508	25.019	0.119	24.0017	0.8983
DATA	95.09	95	1.43	0.7403	0.6897	0.7617	0.6683	0.7546	0.6754	0.7364	0.6936	0.761	0.669	0.6828	0.7472
	95.09	100	0.055	0.0414	0.0136	0.0429	0.0121	0.042	0.013	0.0411	0.0139	0.0425	0.0125	0.032	0.023
	139.78	110	29.775	29.6815	0.0935	29.6868	0.0882	29.6849	0.0901	29.7499	0.0251	29.6825	0.0925	29.1508	0.6242
IBM	139.78	140	0.86	0.8785	0.0185	0.9108	0.0508	0.9	0.04	0.9106	0.0506	0.9023	0.0423	0.8161	0.0439
	139.78	142	0.18	0.1645	0.0155	0.1862	0.0062	0.1787	0.0013	0.166	0.014	0.2377	0.0577	0.1024	0.0776
	34.04	25	8.85	9.0141	0.1641	9.0159	0.1659	9.0153	0.1653	9.0229	0.1729	9.0145	0.1645	8.0187	0.8313
YHOO	34.04	34	0.3	0.2699	0.0301	0.2775	0.0225	0.275	0.025	0.2677	0.0323	0.2662	0.0338	0.258	0.042
	34.04	35	0.06	0.0772	0.0172	0.0778	0.0178	0.0776	0.0176	0.071	0.011	0.0785	0.0185	0.0632	0.0032
Average	81.365	75.916	6.65	6.6142	0.1064333	6.626691	0.110225	6.622733	0.10861666	6.618733	0.10361666	6.626325	0.11270833	6.330875	0.31965833

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The Fourth Topic
Conclusions and Recommendations

4-1 Conclusions:

The following conclusions have been reached:

1. The Heston model almost pricing all options close to its market price, but this model depends on the accuracy of its prices on the initial values of the parameters it consists of, as well as the complexity that accompanies its application.
2. An Increase in the average of the (k) Parameter will leads to a decrease in the average price of the options. This means that the implied price volatility of the share price will settle towards stability at the average level of the volatility. This will lead to a decrease in the volatility curve. indicating a decrease in the implied volatility and the price of all options at all strike prices.
3. When the Mean reversion rate (k) is increased, this will result in lower implied volatility and an increase in the value of k. The change in the price of the option will be greater whenever the option is moved from the equal option to the losing option.
4. When the value of (θ) decreases, the value of implied volatility will decrease, and therefore the option price will fall, which will be lower in the case of equal options.
5. 5 - When the value of the average variance (v_0) decreases, this will lead to a decrease in the prices of the winning options as well as to the equal and losing options, but to a lesser degree in the case of the winning options.
6. The lower the value of the volatility of volatility (σ), the lower option prices will be due to the lower risk to the option editor.

2.4 Recommendations:

This section reviews the most important recommendations in this research:

1. Encourage option market traders to pay close attention to the spot prices of the underlying stocks of the option contract as they have a significant impact on the returns that option buyers can receive.
2. The research recommends that researchers undertake more studies and researches that deal with the pricing of options to fill the large knowledge gap at the local and regional level in this important field of quantitative financial management, as it represents a breeding ground for researchers in the field of modern financial management, which is still controversial to many researchers, To date, no model has been agreed upon, in which investors are assisted in making the investment decisions.
3. Make an extensive studies of the Heston model, whether in pricing options or pricing financial assets in general, because of its high capacity to produce prices close to market prices.
4. Urge researchers to do more studies and research in the field of financial engineering and try to contribute to this field of knowledge because of its great importance in the field of hedging and risk management.
5. Try to study and analyze the models of pricing options with jumps, because it simulates the reality in practice at times, especially in times of crisis and disasters and wars.
6. Urge researchers to deal with computer and information programs in solving problems in the field of financial management because of their great potential and not only adhere to the statistical methods.

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The References :

A-Books

1. Gatheral, Jim & Lynch, Merrill, "Stochastic Volatility and Local Volatility", Case Studies in Financial Modelling Course Notes, Courant Institute of Mathematical Sciences, 2002.
2. Walmsley, Julian, "The Foreign Exchange and Money Markets Guide", 2nd edition, John Wiley and Sons Inc., 2000.

B- Journals & Periodicals :

1. Bauer, Rudolf, "Fast Calibration in the Heston Model", Diplomarbeit Thesis Submitted to VIENNA University of Technology, 2012.
2. Gauthier, Pierre & Rivaille, Pierre – Yves H., "Fitting the Smile – Smart Parameters for SABR and Heston", 2009.
3. Gong, Hugh, Dunn, Robin, Hauser, Paloma, and Seibold Tom, 2014.
4. Heston, Steven L., "A Closed- Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options", Yale University, Vol. 6, No. 2, the Review of Financial Studies, 1993.
5. Homescu, Cristian, "Local Stochastic Volatility Models: Calibration and Pricing", 2014.
6. Kangro, Iimars & Kalis, H., "Simple methods of engineering calculation for Solving heat transfer problems", Journal of Mathematical Modelling and Analysis, Vol.8, No.1, 2003.
7. Kangro, R., Parna, K., and Sepp, A., "Pricing European-Style Options Under Jump Diffusion Processes with Stochastic Volatility : Applications of Fourier Transform", Acta et Commentationes Universities Tartuensis de Mathematica, Vol.8, 2003.
8. Karaflos, Dimosthenis, "Application of the Heston's Stochastic Volatility model on the Greek Stock Market", 2016.
9. Mitra, Sovan, "A Review of Volatility and Option Pricing", Preprint Submitted to Elsevier, 2009.
10. Nykvist, Johan, "Time Consistency in Option Pricing Models", Stockholm, 2009.
11. Orosi, Greg, "Empirical Performance of Option Pricing Models with Stochastic Local Volatility, Research, India Publications, Mathematics Modelling and Applied Computing, Vol. 1, No. 1, 2010.
12. Poon, Ser- Huang, "The Heston Option Pricing Model", Springer- Verlag London Ltd., 2011.

C-Thesis & Dissertations :

1. Alma, Helgadottir Dogg & Ionescu Luiza, "Option Pricing Within the Heston Model", Master Thesis, AARHUS University, 2016.
2. Crisostomo, Ricardo, "An Analysis of the Heston Stochastic Volatility Model: Implementation and Calibration using Matlab", in partial fulfillment of the requirements of an MSc degree in mathematics at the National Distance Education University (UNED), Madrid, 2014.
3. Ladokhin, Sergiy, "Volatility Modeling in financial markets", Master thesis, University of Amsterdam Faculty of Sciences, Business Mathematics and Informatics, 2009.
4. Moodley, Nimalin, "The Heston Model: A Practical Approach with MATLAB Code", An Honours Project Submitted to the Faculty of Science, University of

ISSN : 1813-6729 http://doi.org/10.31272/JAE.42.2019.119.1

the Witwatersrand, Johannesburg, South Africa, in Partial Fulfillment of the requirements of the Bachelor in Advanced Mathematics of Finance, 2005.
5. Nguyen, Andreas & Solvang, Joachim Baklid, "Empirical Comparison of Alternative Option Pricing Models", Master Thesis, MSC Applied Economics and Finance at Copenhagen Business School, 2013.
6. Yang, Yuan, "Valuing a European Option with the Heston model", Thesis Published in Rochester Institute of Technology to the School of the Mathematical Sciences in Partial Fulfillment of the requirements for the degree of Master of Science in the subject of Applied and Computational Mathematics , 2013.
7. Ye, Ziqun, "The Black Scholes and Heston Models for Option Pricing", A Thesis Presented to the University of Waterloo in Fulfillment of the thesis requirement for the degree of Master of Mathematics in Statistics, Waterloo, Ontario, Canada, 2013.
8. Ruf, Johannes, "Local and Stochastic Volatility", Oxford-Man Institute of Quantitative Finance, 2012.

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