Expected Travel Distances in Distribution Problem

م. خولة خالد إسماعيل*

Abstract:

There are two ways to estimate the expected length of travelling distances through number 'n' of random points distributed over some area, theoretical methods and simulation way. The comparison had been made firstly, the results of the above methods for several position of the workplace relative to the centre of area. Secondly the comparison had been made for the simulation results themselves, but for different shapes of area (i.e., square, circle and rectangle).

Introduction:

The estimation of travelling distance is a distribution problem. To establish what transport requirement exist in throughout the region to transport, for example, the workers to and from the workplace by vehicle which are provided the Services Department, travelers having to visit 'n' workers, he starts from the vehicle's base and need to visit each of the other 'n-1' workers only once and then return to the vehicle's base.

In some cases, the Department itself provides both of the vehicles and the drivers, however, there is a crash in the afternoon when the workers are being taken home from the workplace and they are often could be more than thirty workers. This could be therefore (the vehicles type) be used in range from small cars (taxis) through to large buses.

The cost of travelling between any pair points (expressed in terms of distances times of monetary expenditure). Say from point i to another one j, is given as cij in a cost matrix c. Although the cost of operating or hiring the different sorts of vehicles vary greatly, but it is necessary to design such a route through the 'n' workers that would minimize the total cost of the tour.

Hence, there are two ways to estimate the travelling distance, theoretical methods and the simulation way, moreover, two theoretical methods have been proposed are BHH method and EC method. Also we had been interested with the shape of the area, so we compared the results for the square area with circular and with the rectangular areas.

The Aim of the Project

This project has two steps. Firstly to use two theoretical methods to estimate travelling distance and to compare the results to each method with the results of the simulation method for the same shape of the area, and recommended which one is better.

---

* مدرس/ جامعة الموصل/كلية الإدارة والإقتصاد/قسم الإحصاء

مقبول للنشر بتاريخ 2011/7/24

Secondly, to determine whether the tour distance varies much with the shape of the area, for instance, a square, a rectangular area and a circular area. In addition to determine which the tour distance varies much with the direction of the workplace from the centre of the area.

1- Methods Proposed for Estimating Travel Distances

We consider the problem of estimating the expected length of the travelling distances through number 'n' of random points distributed over some areas.

1.1 Bearwood, Halton and Hammbersly (BHH) Method [3]:

BHH consider the problem of calculating the expected length of the shortest route in arbitrary region of size 'A' containing the n points, by dividing 'A' into K disjoint sub regions. Then the total travel length can be calculated as equal to the summation of the total travel distance in each subregion and the distances need to join the tours in the different subregions together. Finally, they show mathematically "the length of travelling" distance would be:

$$L = k \sqrt{n}\,\sqrt{A} \qquad \text{if n is large}$$

When:
L = the length of travelling distance.
n: the number of points.
A: the size of the area.
K: 0.75 (constant)

Because of the number of workers in our problem will in practice often be small then we could estimate the expected travel distances as:

$$L = k\,\sqrt{n+1}\,\sqrt{A} \quad \ldots(1)$$

if n is small (regarding the workplace as an additional random points).

The shape of the area does not affect the results which are obtained by this formula, and it is assumed that the vehicles in based at the workplace and that workplace is inside the area being served.

1.2 Eilon & Christoferdes (EC) Method [1][2]:

EC consider how to estimate the total travel distance involves in making deliveries from a specified based to 'n' customers at specified location within an area of size A in tours of Dr, the sum of the radial distances between the workers and the workplace. If ξ represents the deliveries that can be transport on a single vehicle trip because of limited vehicle capacity, then the total travel distance is:

$$L= C_1\, Dr/\xi + C_2 \sqrt{Dr}\, \sqrt[4]{A}$$

On the basis of simulation runs for the case when the workers are randomly located within a square, EC suggest taking C_1= 1.8, C_2= 1.1 (or a small, but in determinate amount less, if the vehicle base is within the square but not at the middle).

The relationship between L and Dr with ξ as a parameter will be a linear relationship between L and Dr, if ξ has a low value, because of the first term of the equation predominate on the other hand for large value of ξ, the second term of the equation predominates.

We could estimate the expected travel distances in our problem by considering the capacity of the vehicle ξ to be equal number of workers 'n' times the expected distance from the vehicle base to workers (d), then:

$Dr = n \times d$

The expected distance from the vehicle base, then become

$L = 1.8\ d + 1.1\ \sqrt{nd}\ \sqrt[4]{A}$ …(2)

The condition on using this formula is that vehicle must be based at the workplace. Also the values for C_1 and C_2 assumed that the workplace is within the area being served. The formula is not exact even for large n.

Thus one has to determine the expected straight – line distance between two points. EC determine the expected straight – line distance between two points for a circular area and a rectangular area. R is a distance between a fixed point P (which is the position of the workplace) and the centre of circle. Then the expected distance E(d) between P and Q (a random point in the circle, which is the position of the worker). When R= 0 which had been derived is:

$\left| E(d) \right|_{R=0} = 2/3\ a$ …(3) when a is the radial of the circle

In addition to the expected distance between the fixed point P with co–ordinates (X_0, Y_0) inside or outside the rectangular area (measuring a by b) and Q be a random point inside the area with co-ordinates (X, Y) is at given distance R from the centre of a rectangular area, then:

$\left| E(d) \right|_{R=0} = 1/3\ r_1 + a/24\ (a/b \ln H_1 + b^2/a^2 \ln H_2)$

When:

$r_1 = \sqrt{A^2 + B^2}$ $r_2 = \sqrt{A^2 + Y_0^2}$

$r_3 = \sqrt{X_0^2 + B^2}$ $r_4 = \sqrt{X_0^2 + Y_0^2}$

$H_1 = B+r_1 / -Y_0 + r_2$ $H_2 = A+r_1 / -X_0 + r_3$

$H_3 = B+r_3 / -Y_0 + r_4$ $H_4 = A+r_2 / -X_0 + r_4$

$A = X_0 = ½\ a$ when r_1, r_2, r_3, r_4 are the distance from P to the four $B = Y_0 = ½\ b$ corners of the rectangle

$r_1 = r_2 = r_3 = r_4$

$H_1 = H_3$ and $H_2 = H_4$

Although we haven't any formula for a square area, but we could derive it by following formula for a rectangular area which is related with same of the above conditions.

$E(d)_R = 1/6\ ab[4AB\ r_1 + 4(A+2R)B\ r_3 + A^3 \ln H_1 + 2B^3 \ln H_2 + (A+2R)\ln H_3$ … (4)

But here, when $A = 1/2\ a – R$

$X_0 = 1/2\ a + R = A + 2R$

$Y_0 = B = ½\ b$

$r_1 = r_2 = r_3 = r_4$, $H_2 = H_4$

Now, by substituting b = a [that means the square area with side 'a'] and R= 0, we can find the expected distance between the workers and the workplace which is at the centre of square as:

$\left| E(d) \right|_{R=0} = 0.383\ a$ … (5)

The formulas below given the expected distance between the workplace and the worker, if the workplace at different distances from the centre of square.

$\left| E(d) \right|_{R=1/2a} = 0.593\ a$ if P is at the middle side of square

$\left| E(d) \right|_{R=a} = 1.043\ a$ if P is outside of square

$\left| E(d) \right| = 0.765\ a$ if P is at the centre of square

If we have real data in true circular , rectangular and square area , we use the formulas (3) , (4) and (5) respectively to decide how to calculate the size of the

area over which the workers are located and the distance of the workplace from the middle of this area .

1.3 Some Simulation Results [4][5]:

Simulation method is probably much more widely used in practice than is evident from availability of suitable data .

The result is likely to be affected by the initial position of the workplace, the exercise needs to be repeated for several initial condition. This method is equally valid for the assumption that all 'n' workers are served by a vehicle in one single tour and for the case where several tours need to be designed for each period.

Hence, to examine the two theoretical methods, the values of the optimum tour lengths are required. These were calculated by simulating workers distributions and then calculating the optimum tour lengths using the 3 – optimal methods which made the basis of the program used by the project.

Now, consider the case where the workers are randomly distributed throughout a unit square area of side 'a' with the workplace at the different positions from the centre of the area.

Figure (1) represents the several positions of the workplace relative of the centre of square. In case (a) the workplace is at the centre of square, co-ordinates (0.5, 0.5)

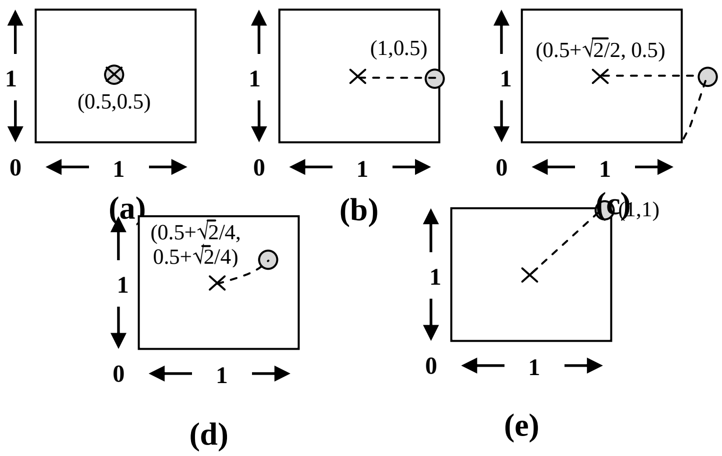

Figure (1) represents the several positions of the workplace in unit square area.

In case (b) the workplace is from the middle of square a (1.0, 0.5), in case (d) it is at (0.5+√2/4, 0.5+√2/4), in both cases the distance between the workplace and the centre of square equal to (0.5) unit, but in different direction. Similarly in case (c) the workplace is at (0.5+√2/2, 0.5) outside the area, and in case (e) the workplace at the corner of square. Clearly the distance for these two cases between the workplace and the centre of square is the same but a gain in different direction.

Following is the algorithm of calculating the mean travelling distance by using simulation :

1- Determine the co-ordinate of workplace location (P_0) .

2- Generate the co-ordinate the points (p_1 , p_2 , … pn) which represent the location of the workers at square area with side 'a' , when n represents number of workers .
3- Find the matrix of distance (d) between every two points (0,1,2,..n) with (n +1 , n +1) dimension by using :

dij = $\sqrt{(xi - xj)^2 + (yi - yj)^2}$

4- Determinding vector L which is the vecor of linking each point with the nearest one .
5- Start the journey from (P_0) , put i = 0 , j =0 , all vector L elements equal zero .
6- Let x is th largest value in row i of d matrix , m=0 .
7- If dij $\leq$ x and Lj = 0 when i $\neq$ j , then dij = x , m = j .
8- Let j = j + 1 .
9- If j $\leq$ n go to (7) .
10-Let Li = m , i= m , dij = Dk , k = (1,2,…n+1) .
11-If i $\neq$ 0 go to (6) .
12-Calculate D = $\sum$Dk /(n+1) .
13-end.

We can use the same algorithm above for circular and regtangular area .

From Table (1), we can see the results which we got from the computer. ECM have also done similar simulation for seven workers and we improved to twenty workers.

If we compare these results, we see that in case when the workplace at the centre of square, travelling distance is smallest. In case (b) and (d) the distance between the workplace and the centre of square is equal to 0.5 unit and if we look at the results which are given in Table (1) we will find the average travel distances are approximately equal, for example, for 5 workers equal to 2.38 in cases (b) and (d) equal to 2.41. Similarly in cases (c) and (e), if we compare the average travel distances, for example, for twenty workers, we will find, in case (c) this equals 4.25 and in case (e) 4.23.

Thus we can say, that the average tour length appear almost independent of direction to which workplace is displaced from the middle of square.

Table (1)
Simulation results of mean tour length in a unit square area

No. of workers	Workplace at				
	(a)	(b)	(c)	(d)	(e)
2	1.2	1.79	2.16	1.8	2.16
3	1.49	1.94	2.3		
4	1.75	2.14	2.495		
5	2.055	2.38	2.71	2.41	2.75
6	2.38	2.64	2.96		
7	2.525	2.69	3.0		
8	2.725	2.81	3.10		
9	2.88	3.03	3.34		
10	3.0	3.11	3.43	3.08	2.38
15	3.45	3.46	3.75		
20	3.90	3.95	4.25	3.96	4.23

2-Comparison of Theoretical Methods with Simulation Results:

Table (2) gives the expected length of tour in unit square area, by BHH method. Clearly the distance is the same for all cases (for any position of the workplace in the area), because the formula L= k √n+1 √A does not depend on the workplace position (vehicle's base), but it is assumed to be inside the~~ are~~a. ~~So~~ the expected tour length is the same when the workplace at the middle of square or at the middle of a side or at a corner.

Obviously, these results do not compare well with simulation ones, which are given in Table (1) for any case because they depend on the only one factor which is the size of the area and not on the position and the direction of the workplace.

Table (2)
Expected travel distance for several positions of the workplace, using BHH method.

No. of workers	Workplace at			
	Middle square	Middle side	The corner	Outside
1	1.061	The same as first column	The same as first column	The same as first column
2	1.30			
3	1.50			
4	1.68			
5	1.84			
6	1.99			
7	2.12			
10	2.5			
15	3.00			
20	3.5			

On the other hand, Table (3) gives the expected tour length by EC method (L = 1.8 Dr/n + 1.1 ^{4}√A √Dr) for three positions of the workplace in unit square area, at the middle of square, at the middle of a side and at corner.

Hence, if we take the results from Table (1) for the same position of the workplace as the positions assumed for EC method, we find that the results by EC method, when the workplace at the middle of square, are not too bad. If n≥ 5 (n: the number of workers), but if we compare the results using the EC method when the workplace at the middle of a side or at corner we find they do not compare well with the simulation results.

Table (3)
Expected tour length for several position of the workplace using EC method.

No. of workers	Workplace at		
	Middle square	Middle side	The corner
1	1.37	1.92	2.34
2	1.65	2.27	2.74
3	1.87	2.54	3.05
4	2.05	2.76	3.30
5	2.21	2.96	3.53
6	2.36	3.14	3.73
7	2.49	3.31	3.92
10	2.84	3.75	4.42
15	3.33	3.35	5.11
20	3.43	4.86	5.68

For example, with twenty workers, the expected tour length by EC method is equal to 4.86, when the workplace at the middle of a side and by simulation method, it is equal to 3.95, and similarly when the workplace is at the corner of the square.

Hence, we can say, from a comparison of the estimates of tour length using EC method for the above positions with the simulation ones, that the EC method is not too bad if the workplace is at the middle of square and when the number of workers is equal to or greater than five, but does not depend on the direction in which workplace is placed.

3-Effect of Area Not Being Square:

The simulation results shown in Table (1) are for a square area and for different position of the workplace from the centre of square. We must therefore check to see what errors are caused if the area is non – square.

Therefore, we should ideally arrange for our random points in a unit circle to have “the same sort of spread as our random points in a square”, and finally, we can get the results of expected travel distance for the circular area which is set out in Table (4), moreover, we have to choose the same distances between the workplace and the centre of circle as the square.

Table (4)
Simulation result of mean travelling distance in a unit of circular area .

No. of workers	Workplace at			
	(a) The centre co-ordinates (0,0)	(b) 0.5 from the centre co-ordinates (0.5,0)	(c) 0.707 from the centre co-ordinates (0.707,0)	(d) 1 from the centre co-ordinates (1.0,0)
2	1.16	1.75	2.12	2.68
5	2.0	2.32	2.66	3.20
10	2.91	3.02	3.34	3.88
20	3.84	3.80	4.14	4.69

If we compare the result in Table (4) in cases (a), (b), (c) and (d), when the distance of the workplace from the centre of circle is approximately the same as in the cases (a), (b), (c) and (e) of the square respectively, we will find the differences are not very much.

But, we had arranged for our random points in a unit rectangle to have the same of sort of spread as our random points in a square and we had taken the distance of the workplace from the centre of rectangle the same as the square.

Table (5)
Simulation results of mean travelling distance in a unit rectangular area .

No. of workers	Workplace at		
	(a) The centre co-ordinates (1,0.25)	(b) 0.5 from the centre co-ordinates (1.5,0.5)	(c) 0.5 from the centre co-ordinates (1.0,0.75)
2	1.62	2.16	2.15
5	2.76	2.93	3.13
10	3.60	3.63	3.89
20	4.45	4.46	4.84

Table (5) gives the expected travel distance in unit rectangular area for three positions of the workplace at the centre of rectangle. In case (a) the workplace at the centre, but in both cases (b) and (c) the workplace at (0.5) from the centre but in different direction.

Now if we compare these results with the simulation ones of the square, we find the difference to be much greater.

Conclusions and Recommendations

1. **The simulation results show that the average tour length appears almost independent of direction in which workplace is displaced from the middle of square.**
2. **The results by EC method show that when the workplace at the middle of square, are not too bad, if n ≥ 5 (n = number of workers). Moreover, these results show that when the workplace at the middle of a side or at a corner, they do not compare well with the simulation results.**
3. **EC method does not depend on the direction in which workplace is placed.**
4. **The results by BHH method do not compare well with simulation ones, for any position of the workplace because this method depends on the only one factor which is the size of the area and not on the position and the direction of the workplace.**
5. **The simulation results show that the mean tour length for the square area and for circle area almost the same and the differences are not vary much for all cases (the several position of the workplace relative to the centre). But the simulation results show that difference between the mean tour length for the square area and for rectangle area to be much greater. That means that method seems to work well if the area is not too alongated.**

References

1. **David, P. Chu, 2006, “The Distance between Random Points in Two Rectangular Cities”, Communications in Statistics – Simulation and Computation, Vol. 35, Page 257 – 276.**
2. **Daskin. M, 2008 , What Should you Know about**

Location Modelling , Wiley Interscience (www. Intersience.wiley.com)

3. **Diana, M., 2003, "Modelling Versus Simulation: Two Approaches for the assessment of the Service Quality Cost Relationship in innovative public transport systems" Paper presented at the joint ECTRI-FERSI Young Researcher Seminar, Lyon – Bron.**
4. **Eilon, Watson – Gandy, Christofides, 1971, "Distribution Management: Mathematical Modelling and Practical Analysis", Griffin, London.**
5. **Gunag-B , 2009 , Fundamentals of Warehousing Problems , Second Edition , Bright pen publishing , UK .**
6. **John, E. Sussams, 1971, "Efficient Road Transport Scheduling: A Practical Approach to Vehicle Routing", Gower, London.**
7. **John, E. Sussams, 1969, "Industrial Logistic: Application of Scientific Techniques to the Practical Problems", Gower, London.**
8. **Rajagopalan–H, Saydam–C. , 2006 , A Minimum Expected Response Model : Formulation , Heuristic Solution and Application , International Journal of Socio-Economic Planning Sciences.**
9. **شبكة المعلومات الدولية**

Chapter 5 : Distance analysis I and II
From www.icpsr.umich.edu/crimestat-files-crimestatechapter,5.PDF

•••

•••

••••••••••••••••